RECITATION 7: Lines and Planes

Goals: In this project, we imagine we are working with an expensive material, so it is important to create designs that uses it frugally. We'll use what we know about planes, normal vectors, and parametric equations to solve two problems.

Problem 1

Californium-252 is a radioactive isotope that’s used in treating cervical and brain cancer and also in explosives detectors at airports. It is the most expensive chemical substance in commercial use, with a production cost of US$300 per microgram (as of 2009). The costs of transporting and safely storing and handling the isotope add to the production costs.

For an application, you need to connect a node at the point \( P = (-1, -6, 3) \) (coordinates are in inches) to any point on the surface \( S \) defined by the equation \(-3x + 8y + 5z = 38\) with a thin strand of \( ^{252}\text{Cf} \) at a density of 50 micrograms per inch. Your team's goal is to design this strand in a way that minimizes the cost. This will involve finding the point on the surface that is closest to \( P \).

Work with your group members to complete these steps:

G1. Write the coordinates of a vector normal to the surface \( S \). Call this vector \( \vec{n} \).

G2. Find the parametric equations of a line that goes through \( P \) in the direction of the normal vector \( \vec{n} \). Why is this line important?

G3. Combine the parametric equations you found in G2 with the equation \(-3x + 8y + 5z = 38\) to create an equation with a single variable \( t \). Solve that equation for \( t \).

G4. Find the point \( Q \) on the surface where the strand of californium will connect. Then find the length of \( PQ \).

G5. Calculate the cost of the strand.

Problem 2

Two plane surfaces, whose equations are given by \( x + y + 2z = 6 \) and \( x - y + 3z = -1 \) intersect in a line. The seam where the planes intersect will be coated with a thin strand of \( ^{252}\text{Cf} \) between the points determined by \( x = 1 \) and \( x = 5 \) (as shown to the right.)

Your team’s goal is to find the length of the coated seam. (See the next page.)
Spend no more than eight minutes working these three problems, then reunite with your group members:

I. The seam starts at $x = 1$ and ends at $x = 5$. Why isn't the length of the seam 4 units?

II. One endpoint occurs where $x = 1$. Use the two plane equations to find the values of $y$ and $z$ so that you have the coordinates of the endpoint.

III. Now find the coordinates of the other endpoint (where $x = 5$).

IV. Based on the two endpoints you found, find the length of the seam.

Work with your group members to complete these steps:

G6. Find a vector connecting the two endpoints you found above.

G7. Find the cross-product of the normal vectors to each of the planes. The direction (but possibly not the magnitude) of this cross-product should coincide with the vector you found in the previous step.

G8. Explain why the cross-product of the normal vectors should give you the direction of the seam. You may want to use an illustration to help with this explanation.