RECITATION 3: Graphing Polar Equations

Goals: The goal of this recitation is to increase your confidence in working with the polar coordinate system and graphing functions and inequalities expressed in terms of \( r \) and \( \theta \).

As with previous recitations, you’ll be working in small groups on this worksheet.

Converting between polar coordinates and Cartesian coordinates

Instructions: To convert the point \((r, \theta)\) to a point \((x, y)\), apply the following formulas:

\[
\begin{align*}
x &= r \cos \theta \\
y &= r \sin \theta.
\end{align*}
\]

To convert the point \((x, y)\) to a point \((r, \theta)\), apply the following formulas:

\[
\begin{align*}
r &= \sqrt{x^2 + y^2} \\
\tan \theta &= \frac{y}{x}
\end{align*}
\]

Group Exercise: Convert each of the following equations from polar to Cartesian, or from Cartesian to polar, as indicated.

G1. \(3x^2 + 3y^2 = 12\)

G2. \(3x^2 + 5y^2 = 49\)

G3. \(r = \frac{9}{2 \sin \theta + 5 \cos \theta}\)

Solve the following application:

G4. Two wheels, with radii 5 and 9 respectively, are joined by a belt as shown in the diagram. The distance between the wheels at the nearest point is 6 units. The wheel on the right is turned 0.7 radians clockwise from the position shown. Compute the position, in Cartesian coordinates, where point P comes to rest.
Graphing Functions in Polar Coordinates

We'll use an example to motivate how to graph functions in polar coordinates.

**Example:** Graph the function \( r = -2 \sin 3\theta \).

**Step 1:** We start by looking at the coefficient of \( \theta \)—in this example, it's 3. This indicates that we want to divide each quadrant of the graph into thirds:

![Graph of r = -2 sin 3\( \theta \)](image)

**Step 2:** At each of those dividing lines, we calculate the value of \( r \) based on the function, and label those values on the graph:

![Graph with values labeled](image)

**Step 3:** We plot a point for each of these radii: if the radius is negative, we plot the point on the opposite side. In this example, you'll find that many points “overlap” other points you've already plotted—that won't always be the case.

![Graph with points plotted](image)
Step 4: Finally, we connect the points we plotted, one sector of the graph at a time. For example, between $\theta = 0$ and $\theta = \frac{\pi}{6}$, we connect the point at $r = 0$ to the point at $r = -2$. Since $r = -2$ is negative, this takes place in Quadrant III, even though our angles are in Quadrant I! Then between $\theta = \frac{\pi}{6}$ and $\theta = \frac{\pi}{3}$, we connect the point at $r = -2$ back to the point at $r = 0$. This also takes place in Quadrant III. Then between $\theta = \frac{\pi}{3}$ and $\theta = \frac{\pi}{2}$, we connect the point at $r = 0$ to the point at $r = 2$. Since $r = 2$ is positive, this takes place in Quadrant I:

![Graphs showing steps of connecting points](image)

Step 5: If we continue this process all the way around the circle, we get the finished graph:

![Finished graph](image)

Individual Practice: Spend no more than eight minutes graphing these two functions, then rejoin with your group members to check your answers:

I. $r = 1 + 2 \cos(2\theta)$

II. $r = 2 \sin \theta - 1$

Group Exercises: Graph these equations. There are a few extra details to figure out and pay attention to:

G5. $r^2 = \sin(2\theta)$. As you're graphing, discuss what effect $r^2$ has on this equation: What happens when $\sin(2\theta)$ is positive? What happens when it's negative?
G6. \( r = 2 \cos(3\theta) \) on the interval \( \frac{\pi}{3} \leq \theta \leq \frac{3\pi}{2} \). As you're graphing, discuss what effect the interval has on the shape of the final graph.

G7. \( r^2 - 3r + 2 = 0 \)

**More Group Exercises:** Shade in the solution sets of these inequalities:

G8. \( 1 \leq r \leq 3 \) and \( \frac{\pi}{4} \leq \theta \leq \pi \).

G9. \( -2 \leq r \leq -1 \) and \( \frac{\pi}{2} \leq \theta \leq \frac{5\pi}{4} \).

G10. \( -1 \leq r \leq 2 \) and \( \frac{\pi}{3} \leq \theta \leq \frac{2\pi}{3} \). If you're having trouble with this, consider these separate cases: What does the graph look like for positive values of \( r \) (\( 0 < r \leq 2 \))? What does the graph look like for negative values of \( r \) (\( -1 \leq r < 0 \))? What does the combined graph look like?