Math 1025: Elementary Calculus
Sec. 1.3: Linear Functions and Models

I. Linear Function
A. Definitions
1. A linear function is one that can be written in the form $f(x) = mx + b$ (function form) or $y = mx + b$ (equation form) where $m$ is the slope and $b$ is the $y$-intercept (both are real numbers).

2. The slope, $m$, of a line through points $(x_1, y_1)$ and $(x_2, y_2)$ is

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$ 

3. Parallel lines: lines are parallel (//) if they have the same slope.
   Ex: $y = 2x - 1$ and $y = 2x + 3$

4. Lines are perpendicular (⊥) if one has the slope $m$ and the other has the slope $-\frac{1}{m}$.
   Ex: $y = 2x - 1$ and $y = -\frac{1}{2}x + 3$

B. Line Forms
1. Point-Slope Form: $y - y_1 = m(x - x_1)$ where $(x, y)$ is a point on the line and $m$ is the slope.

2. Slope-Intercept Form: $y = mx + b$ where $m$ is the slope and $b$ is the $y$-intercept.

3. General Linear Form: $Ax + By + C = 0$, where $A, B, C \in \mathbb{R}$ and $A$ and $B$ both not equal to zero.

C. Examples
1. Slope Examples: Find the slope of the line through the points given and sketch the line:
   a. $(1,2)$ and $(4,6)$:

   $$m = \frac{y_2 - y_1}{x_2 - x_1}$$

   This slope _______________ or _______________.

   b. $(1,2)$ and $(-3,4)$:

   $$m = \frac{y_2 - y_1}{x_2 - x_1}$$

   This slope _______________ or _______________.
Find the slope of the line through the points given and sketch the line:

c. (1,2) and (3,2):

\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 2}{3 - 1} = 0 \]

\[ \text{lines have slope } m = 0. \]

The equation of a horizontal line with \( y \)-intercept = \( c \) is \( y = c \).

The equation for the line through our points is ________________.

d. (1,2) and (1,−5):

\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - 2}{1 - 1} \text{ is undefined} \]

\[ \text{lines have undefined slope.} \]

The equation of a vertical line with \( x \)-intercept = \( c \) is \( x = c \).

The equation for the line through our points is ________________.

2. Line Examples:
   a. Find an equation for the line in point-intercept form and slope-intercept form
      1.) slope = 8; a point on the line is (4,−1).

      \[ y = mx + b \]
      \[ 1 = 8(4) + b \]
      \[ b = 1 - 32 = -31 \]
      \[ y = 8x - 31 \]

      \[ y = mx + b \]
      \[ -1 = 8(4) + b \]
      \[ b = -1 - 32 = -33 \]
      \[ y = 8x - 33 \]

      2.) slope = \( \frac{-1}{3} \); the line passes through the origin.

      \[ y = mx + b \]
      \[ 0 = \frac{-1}{3}(0) + b \]
      \[ b = 0 \]
      \[ y = \frac{-1}{3}x \]

      \[ y = mx + b \]
      \[ 0 = \frac{-1}{3}(0) + b \]
      \[ b = 0 \]
      \[ y = \frac{-1}{3}x \]

      3.) line passes through the points (−2,−4) and (1,−1).
b. State the slope and y-intercept of the line and graph.

1.) \(6x - 5y - 20 = 0\)

2.) \(4x + 8 = 0\)

3.) \(7y - 21 = 0\)