I. Review
A. Polynomial in $x$

A polynomial in $x$ is an algebraic expression of the form

$$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_3 x^3 + a_2 x^2 + a_1 x + a_0$$

where $a_n, a_{n-1}, a_{n-2}, \ldots, a_3, a_2, a_1, a_0$ are real numbers, $a_n \neq 0$, and $n$ is a nonnegative integer.

The degree of the polynomial is $n$.

$a_n$ is the leading coefficient.

$a_0$ is the constant term.

B. Polynomials in Several Variables

A polynomial in two variables, $x$ and $y$, contains the sum of one or more monomials in the form $a x^n y^m$. The constant, $a$, is the coefficient. The exponents, $n$ and $m$, are whole numbers.

The degree of the monomial, $a x^n y^m$, is $n+m$.

The degree of the polynomial is in two variables is the highest degree of all its terms.

C. Domain of Polynomials

Polynomials are defined for all real numbers $x$, i.e., its domain is all real numbers, $\mathbb{R}$.

II. Rational Expressions
A. Definition

A rational expression is the quotient of two polynomials. It is not defined where the denominator equals zero.

$$\text{rational expression} = \frac{\text{polynomial}}{\text{polynomial}}; \text{denominator} \neq 0$$

B. Domain of Rational Expressions

1. The domain of a rational expression is the set of real numbers, except for the numbers that make the denominator zero.

2. Examples: Find the domain of the following:

   a. $\frac{x^2 + 3x + 1}{x - 3}$
      Domain: __________________________

   b. $\frac{2x - 1}{x^2 - 64}$
      Domain: __________________________
C. Simplifying Rational Expressions
1. Definition
   A rational expression is simplified if its numerator and denominator have no common factors.
2. Method
   a. Factor numerator and denominator completely.
   b. Cancel any common factors.

2. Example: Simplify the following and state its domain
   \[
   \frac{x^2 - 10x + 25}{x^2 - 25} =
   \]
   Domain: ____________________

D. Multiplying Rational Expressions
1. Method
   a. Factor numerator and denominator completely.
   b. Cancel any common factors.
   c. Multiply remaining factors using \( \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \).

2. Example: Simplify the following and state its domain
   \[
   \left( \frac{x^2 - 4}{x^2 + 3x - 10} \right) \cdot \left( \frac{2x - 4}{x + 2} \right) =
   \]
   Domain: ____________________

E. Dividing Rational Expressions
1. Method
   a. Invert the denominator.
   b. Multiply using technique above.

2. Example: Simplify the following and state its domain
   \[
   \left( \frac{2x^2 - 2x - 24}{2x^2 + 9x + 7} \right) + \left( \frac{x^2 - 16}{x^2 + x} \right) =
   \]
   Domain: ____________________
F. Adding and Subtracting Rational Expressions

1. Method:
   a. Find Least Common Denominator (LCD) of the rational expressions.
      1) Factor each denominator completely.
      2) List the factor of the 1st denominator.
      3) Add to the list in step 2 any factors of the 2nd denominator that do not
         appear in the list.
      4) LCD is the product of these factors.
   b. Rewrite all rational expressions in terms of the LCD.
      \[
      \frac{a}{b} - \frac{c}{d} = \frac{ad}{bd} - \frac{bc}{bd} = \frac{ad - bc}{bd}
      \]
   c. Add or subtract numerators, writing answer as one fraction.
   d. Simplify, if possible.

2. Examples: Simplify the following and state its domain
   a. \[
   \frac{3x + 2}{3x + 4} + \frac{3(x + 2)}{3x + 4} =
   \]
      Domain: 
   b. \[
   \frac{8x - 3}{2x + 7} - \frac{4x + 9}{2x + 7} =
   \]
      Domain: 
   c. \[
   \frac{4x}{x^2 - 2x - 24} - \frac{x}{x^2 - 7x + 6} =
   \]
      Domain: 
d. \[ \frac{6x^2 + 17x - 40}{x^2 + x - 20} + \frac{3}{x - 4} - \frac{5x}{x + 5} = \]

Domain: 

III. Complex Rational Expressions

A. Definitions

Complex rational expressions, also called complex fractions, have numerators or denominators containing one or more rational expressions.

B. Examples: Simplify the following and state its domain

1. \[ \frac{1 - \frac{1}{x}}{xy} = \]

Domain: 

2. \[ \frac{x}{x - 2} + 1 = \frac{x}{3} + 1 \]

Domain: 

\[ \frac{x}{x^2 - 4} + 1 \]