I. Factoring Polynomials: Methods
A. Common Factor
1. Method: Look for the Greatest Common Factor (GCF). We are using the distributive property in reverse, $ab + ac = a(b + c)$.

2. Examples: Factor the following:
   a. $6x^3 + 27 = \ldots$
   b. $6x^3 + 27x^2 = \ldots$
   c. $4x^2(3x+1)+5(3x+1)=\ldots$

B. Factor By Grouping
1. Method: Some polynomials initially look like they cannot be factored. But if we group suitable terms together it may be factorable.
   a. Group terms with common factors
   b. Factor out the GCF from the grouped terms.
   c. Factor out the polynomial CF

2. Examples: Factor the following:
   a. $x^3 - 3x^2 + 4x - 12 = \ldots$
   b. $12x^3 + 4x^2 + 15x + 5 = \ldots$

C. Factoring Trinomials $ax^2 + bx + c = (d_1x \pm f_1)(d_2x \pm f_1)$
1. “Trial and Error” Method: Factor trinomial into product of two binomials
   a. Algebraic sign of $c$ determines the middle signs of the binomials:
      If $c$ is negative, signs are different: $(+)(-)$
      If $c$ is positive, signs are the same, same sign as the algebraic sign of $b$:
         $(+)(+) or (-)(-)$
   b. Determine the factors of $ax^2$ and $c$.
      The first term of each binomial will be the factors of $ax^2$.
      The second term of each binomial will be the factors of $c$.
   c. The factors of $ax^2$ and $c$ combine to give the middle term, $bx$. 

2. Examples: Factor the following:
   a. \( x^2 - 7x + 10 = \)
   b. \( x^2 - x - 12 = \)
   c. \( 16x^2 + 8x + 1 = \)
   d. \( 4x^2 + 4x - 3 = \)

D. Factoring Difference of Squares
   1. Method: Look to see if binomial is of the form: \( a^2 - b^2 = (a+b)(a-b) \)
   2. Examples: Factor the following:
      a. \( x^2 - 36 = \)
      b. \( 49x^2 - 25y^2 = \)

E. Factoring Perfect Square Trinomials
   1. Method: Look to see if trinomial is of the forms:
      \( a^2 + 2ab + b^2 = (a+b)^2 \) or \( a^2 - 2ab + b^2 = (a-b)^2 \)
   2. Examples: Factor the following:
      a. \( x^2 + 14x + 49 = \)
      b. \( 4x^2 - 12x + 9 = \)
F. Factoring the Sum and Difference of Two Cubes
   1. Method: Look to see if binomial is of the forms:
      \[ a^3 + b^3 = (a + b)(a^2 - ab + b^2) \quad \text{or} \quad a^3 - b^3 = (a - b)(a^2 + ab + b^2) \]

   2. Examples: Factor the following:
      a. \( x^3 - 27 = \)
      b. \( 8y^3 + 125 = \)

II. Strategy for Factoring Polynomials Completely
   A. Steps
      1. Factor out the GCF, if there is a common factor.
      2. Determine the number of terms in the polynomial and try factoring as follows:
         a. Two Terms (Binomial): Can the binomial be factored using one of the special forms?
            1) Difference of two squares: \( a^2 - b^2 = (a + b)(a - b) \)
            2) Sum of two cubes: \( a^3 + b^3 = (a + b)(a^2 - ab + b^2) \)
            3) Difference of two cubes: \( a^3 - b^3 = (a - b)(a^2 + ab + b^2) \)
         b. Three Terms (Trinomial):
            1) Perfect Square Trinomial: use
               \[ a^2 + 2ab + b^2 = (a + b)^2 \quad \text{or} \quad a^2 - 2ab + b^2 = (a - b)^2 \]
            2) Not a perfect square trinomial: try trial and error method.
         c. Four or More Terms: try factor by grouping
      3. Check to see if any factors with more than one term in the factored polynomial can be factored further. Make sure that the polynomial is factored completely.

   B. Extra Examples: Factor the following:
      1. \( x^2 - 25 = \)
      2. \( x^2 + 25 = \)
      3. \( 9x^3 - 9x = \)
      4. \( x^3 + 2x^2 - 4x - 8 = \)