I. Polynomials of a Single Variable

A. Definition of a Polynomial in \( x \)

A polynomial in \( x \) is an algebraic expression of the form

\[
a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_3 x^3 + a_2 x^2 + a_1 x + a_0
\]

where \( a_n, a_{n-1}, a_{n-2}, \ldots, a_3, a_2, a_1, a_0 \) are real numbers, \( a_n \neq 0 \), and \( n \) is a nonnegative integer.

The degree of the polynomial is \( n \).

\( a_n \) is the leading coefficient.

\( a_0 \) is the constant term.

B. Names of Polynomials

1. Monomial - a simplified polynomial that has exactly one term
2. Binomial - a simplified polynomial that has two terms
3. Trinomial - a simplified polynomial that has three terms
4. Simplified polynomials with four or more terms have no special name.

C. Examples

1. State the degree for the following polynomials:
   a. \( 3x^5 + 2x^4 - x^2 + \frac{1}{2} x + 7 \)  degree:_____
   b. \( 7x^3 - 4x^2 + x - 5 \)  degree:_____
   c. \( -x + 1 \)  degree:_____
   d. \( 13 \)  degree:_____

2. State whether or not the following is a polynomial. If not, state why.
   a. \( 2x^{-5} + 9x^4 - x + 6 \)  ________________________________
   b. \( (x^3 + 3x^2)(2x - 5) \)  ________________________________
   c. \( \frac{1}{x+1} \)  ________________________________
   d. \( x^{\sqrt{2}} + 3x + 4 \)  ________________________________
   e. \( (6x^2 + x - 1) - (2x^2 + 5) \)  ________________________________
   f. \( 5x^4 + x^{\frac{1}{3}} + 4 \)  ________________________________
D. Adding and Subtracting Polynomials
   1. Polynomials are added and subtracted by combining like terms.
   2. Examples: Simplify
      a. \((4x^2 + 9x - 1) + (3x^2 - x + 5)\) = ________________________
      b. \((2x^3 + 7x^2 - 6x - 1) - (4x^2 - 9x + 8)\) = ________________________

E. Multiplying Polynomials
   1. Methods
      a. Product of Two Monomials - use properties of exponents (multiply coefficients and add exponents)
      b. Product of Monomial and a Polynomial That is Not a Monomial – use distributive property of multiplication.
      c. Product of Two Polynomials if Neither is a Monomial – multiply each term of one polynomial by each term of the other polynomial and then combine like terms.
         • Product of Two Binomials – use FOIL method
   2. Examples:
      Simplify the following:
      a. \((-4x^2)(3x^5)\) = ________________________
      b. \((2x^3)(4x^2 - x + 5)\) = ________________________
      c. \((3x^2 + 1)(2x^3 - x)\) = ________________________
      d. \((3x^2 + 4)(2x^3 - x + 5)\) = ________________________
      e. Write a polynomial in standard form that represents the area of the shaded region.
3. Special Products
   a. Product of the Sum and Difference of Two Terms
      1) Rule
         \[(a + b)(a - b) = a^2 - b^2\]
      2) Examples
         Simplify the following:
         a) \[(2x + 3)(2x - 3)\] =
         b) \[(x^2 + y^2)(x^2 - y^2)\] =

         Factor the following:
         a) \[y^2 - 16\] =
         b) \[x^6 - 25y^2\] =

   b. The Square of a Binomial
      1) Rules
         \[(a + b)^2 = a^2 + 2ab + b^2\]
         \[(a - b)^2 = a^2 - 2ab + b^2\]

      2) Examples
         Simplify the following:
         a) \[(x - 5)^2\] =
         b) \[(2x + 3)^2\] =

   c. The Cube of a Binomial
      1) Rules
         \[(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3\]
         \[(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3\]

      2) Examples
         Simplify the following:
         a) \[(x - 2)^3\] =
         b) \[(2y + 1)^3\] =
II. Polynomials in Several Variables

A. Definition

A polynomial in two variables, $x$ and $y$, contains the sum of one or more monomials in the form $ax^ny^m$. The constant, $a$, is the coefficient. The exponents, $n$ and $m$, are whole numbers.

The degree of the monomial, $ax^ny^m$, is $n+m$.

The degree of the polynomial is in two variables is the highest degree of all its terms.

B. Examples

Simplify

1. $(x^3y - 2y) + (3x^3y + 7y + 1) =$

2. $(x^3 - 2y)(3x + 7y^2) =$