Math 1014: Precalculus with Transcendentals  
Ch. 8: Systems of Equations and Inequalities  
Sec. 8.4: Systems of Nonlinear Equations in Two Variables

I. Systems of Nonlinear Equations and Their Solutions
   A. Definitions
      1. A **system** of two nonlinear equations in two variables, also called a **nonlinear system**, contains at least one equation that cannot be expressed in the form \( Ax + By = C \).
      2. A **solution** of a nonlinear system in two variables is an ordered pair of real numbers that satisfies both equations in the system.
      3. The **solution set** of the system is the set of all such ordered pairs.

   B. Examples
      1. Solve by the substitution method: 
         \[
         \begin{align*}
         xy &= 4 \\
         x^2 + y^2 &= 8
         \end{align*}
         \]
2. Solve using the addition method: \[
\begin{align*}
3x^2 - 2y^2 &= -5 \\
2x^2 - y^2 &= -2
\end{align*}
\]

3. Solve \[
\begin{align*}
(x - 1)^2 + (y + 1)^2 &= 5 \\
2x - y &= 3
\end{align*}
\]
4. The sum of two numbers is 20 and their product is 96. Find the numbers.

5. The difference between the squares of two numbers is 5. Twice the square of the second number subtracted from three times the square of the first is 19. Find the numbers.