Polynomials Inequalities

A. Definitions

1. A **polynomial inequality** is any inequality that can be put into the forms
   \[ f(x) < 0, \quad f(x) > 0, \quad f(x) \leq 0, \quad \text{or} \quad f(x) \geq 0, \]
   where \( f(x) \) is a polynomial

   \[ f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_3 x^3 + a_2 x^2 + a_1 x + a_0 \]

   where \( a_n, a_{n-1}, \ldots, a_3, a_2, a_1, a_0 \) are real numbers, \( a_n \neq 0 \), and \( n \) is a nonnegative integer.

2. A **quadratic inequality** is any inequality of the form

   \[ a x^2 + b x + c < 0, \quad a x^2 + b x + c > 0, \]
   \[ a x^2 + b x + c \leq 0, \quad \text{or} \quad a x^2 + b x + c \geq 0, \]

   \( a, b, c \in \mathbb{R}, \quad a \neq 0 \)

B. Solving a Polynomial Inequality

1. Bring all terms to the left, leaving 0 on the right.
2. Set equation = 0 and solve to find “breaking points”.
3. Using the breaking points, divide number line into intervals.
4. Use a test value in each sub-interval to determine if the subinterval satisfies the inequality.

C. Examples:

Solve each of the following and graph the solution set on a real number line. Express each solution set in interval notation.

1. \( x^2 - 4x + 3 < 0 \)
2. \( x^2 + x - 6 \geq 0 \)
II. Rational Inequalities

A. Definition
A rational inequality is any inequality that can be put into the forms
\( f(x) < 0, \ f(x) > 0, \ f(x) \leq 0, \ or \ f(x) \geq 0 \), where

\[ f(x) = \text{rational expression} = \frac{\text{polynomial}}{\text{polynomial}} ; \ denomenator \neq 0 \]

B. Steps
1. Bring all terms to the left, leaving 0 on the right.
2. Combine to create a single fraction (use common denominator)
3. Set equation = 0 and solve to find “breaking points”.
4. Using the breaking points, divide number line into intervals.
5. Use a test value in each sub-interval to determine if the subinterval satisfies the inequality.

C. Examples
Solve each of the following and graph the solution set on a real number line. Express each solution set in interval notation.

1. \( \frac{x+5}{x+2} < 0 \)

2. \( \frac{x+4}{2x-1} \leq 3 \)
3. \( \frac{-3x^2 - 2x}{2x - 7} \geq 0 \)