I. Quadratic Functions

A. Definition

1. A **quadratic function** is a function of the form \( f(x) = ax^2 + bx + c, \ a, b, c \in \mathbb{R}, a \neq 0 \). A quadratic function is a polynomial function whose greatest exponent is 2.

2. The graph of any quadratic function, \( f(x) = ax^2 + bx + c, \ a \neq 0 \), is a **parabola**. If \( a > 0 \), the parabola opens upward; if \( a < 0 \), the parabola opens downward.

   The **vertex** is the turning point of the parabola. The vertex is either the lowest or highest point depending on how the parabola opens.

B. Quadratic Functions in Standard Form

1. The **Standard Form** of a Quadratic Function

   The quadratic function \( f(x) = a(x - h)^2 + k, \ a \neq 0 \) is in **standard form**. The graph of \( f \) is a parabola whose vertex is the point \((h,k)\). The parabola is symmetric with respect to the line \( x = h \). If \( a > 0 \), the parabola opens upward; if \( a < 0 \), the parabola opens downward.

2. Graphing Quadratic Functions with Equations in Standard Form

   To graph \( f(x) = a(x - h)^2 + k \),

   1. Determine whether the parabola opens upward or downward. If \( a > 0 \), it opens upward. If \( a < 0 \), it opens downward.

   2. Determine the vertex of the parabola. The vertex is \((h,k)\).

   3. Find any \( x \)-intercepts by replacing \( f(x) \) with 0. Solve the resulting quadratic equation, \( f(x) = 0 \), for \( x \). The function's real zeroes are the \( x \)-intercepts.

   4. Find the \( y \)-intercepts by replacing \( x \) with 0; i.e., computing \( f(0) \).

   5. Plot the intercepts, vertex and additional points as necessary. Connect these points with a smooth curve that is shaped like a cup. Draw a dashed vertical line for the axis of symmetry.
3. Example
Given \( f(x) = 4(x - 2)^2 + 3 \).

1. Determine the vertex.

2. Determine the \( x \)-intercept(s).

3. Determine the \( y \)-intercept.

4. Determine the line of symmetry.

5. Does the parabola open upward or downward?

6. Determine the domain.

7. Determine the range.

8. Graph
C. Quadratic Functions of the form \( f(x) = ax^2 + bx + c \)

1. The Vertex of a Parabola Whose Equation is \( f(x) = ax^2 + bx + c \)

Consider the parabola defined by the quadratic function \( f(x) = ax^2 + bx + c \). The parabola’s vertex is \( \left( \frac{-b}{2a}, f\left( \frac{-b}{2a} \right) \right) \).

2. Graphing Quadratic Functions with Equations in the Form \( f(x) = ax^2 + bx + c \)

To graph \( f(x) = ax^2 + bx + c \)

1. Determine whether the parabola opens upward or downward. If \( a > 0 \), it opens upward. If \( a < 0 \), it opens downward.

2. Determine the vertex of the parabola. The vertex is \( \left( \frac{-b}{2a}, f\left( \frac{-b}{2a} \right) \right) \).

3. Find any \( x \)-intercepts by solving \( f(x) \) with 0. Solve the resulting quadratic equation, \( f(x) = 0 \), for \( x \). The function’s real zeroes are the \( x \)-intercepts.

4. Find the \( y \)-intercepts by replacing \( x \) with 0; i.e., computing \( f(0) \).

5. Plot the intercepts, vertex and additional points as necessary. Connect these points with a smooth curve that is shaped like a cup. Draw a dashed vertical line for the axis of symmetry.

3. Minimum and Maximum: Quadratic Functions

Consider \( f(x) = ax^2 + bx + c \).

1. If \( a > 0 \), then \( f \) has a minimum that occurs at \( x = \frac{-b}{2a} \).

   This minimum value is \( f\left( \frac{-b}{2a} \right) \).

2. If \( a < 0 \), then \( f \) has a maximum that occurs at \( x = \frac{-b}{2a} \).

   This maximum value is \( f\left( \frac{-b}{2a} \right) \).

In each case, the value of \( x \) gives the location of the minimum or maximum value. The value of \( y \), or \( f\left( \frac{-b}{2a} \right) \), gives that minimum or maximum value.
4. Examples
   a. Given \( f(x) = 2x^2 - 7x - 4 \)
      1. Determine the vertex.

      2. Is the vertex a maximum or a minimum?

      3. Determine the \( x \)-intercept(s).

      4. Determine the \( y \)-intercept.

      5. Determine the domain.

      6. Determine the range.

      7. Graph
b. Given \( f(x) = 5 - 4x - x^2 \)
   1. Determine the vertex.

   2. Is the vertex a maximum or a minimum?

   3. Determine the \( x \)-intercept(s).

   4. Determine the \( y \)-intercept.

   5. Determine the domain.

   6. Determine the range.

   7. Graph
II. Word Problems Involving Quadratic Functions

A. Strategy for Solving Problems Involving Maximizing or Minimizing Quadratic Functions

1. Read the problem carefully and decide which quantity is to be maximized or minimized.

2. Use the conditions of the problem to express the quantity as a function in one variable.

3. Rewrite the function in the form \( f(x) = ax^2 + bx + c \).

4. Calculate \( \frac{-b}{2a} \). If \( a > 0 \), \( f \) has a minimum at \( x = \frac{-b}{2a} \). This minimum value is \( f\left(\frac{-b}{2a}\right) \).

   If \( a < 0 \), \( f \) has a maximum that occurs at \( x = \frac{-b}{2a} \). This maximum value is \( f\left(\frac{-b}{2a}\right) \).

5. Answer the question posed in the problem.

B. Example

1. You have 200 ft of fencing to enclose a rectangular pen. You will use the side of a barn as one side of the pen. Find the length and width of the pen that will maximize the area. What is the largest area that can be enclosed?