I. Domain of a Function
A. Definition
   The domain of a function is the largest set of real numbers for which the function \( f(x) \) is a real number. Exclude real numbers that cause division by zero and real numbers that result in an even root of a negative number.

   - Polynomial functions are defined for all real numbers \( x \), i.e., its domain is all real numbers, \( \mathbb{R} \).
   - The domain of rational functions is the set of real numbers, except for the numbers that make the denominator zero.
   - The domain of radical functions is the set of real numbers, except for the numbers that make the radicand of even roots negative.

B. Examples:
   Find the domain of the following.
   1. \( f(x) = x + 1 \) \( \quad D_f : \quad \) ______________
   2. \( g(x) = \sqrt{x + 1} \) \( \quad D_g : \quad \) ______________
   3. \( h(x) = \frac{3x + 4}{\sqrt{x + 1}} \) \( \quad D_h : \quad \) ______________
   4. \( k(x) = \frac{3}{x + 1} \) \( \quad D_k : \quad \) ______________

II. Combinations of Functions
A. The Algebra of Functions
   Definitions: Sum, Difference, Product, and Quotient of Functions
   Let \( f \) and \( g \) be two functions. The sum \( f + g \), the difference \( f - g \), the product \( fg \), and the quotient \( \frac{f}{g} \) are functions whose domains are the set of all real numbers common to the domains of \( f \) and \( g \) \( \left(D_f \cap D_g\right)\), defined as follows:
   1. Sum: \( (f + g)(x) = f(x) + g(x) \)
   2. Difference: \( (f - g)(x) = f(x) - g(x) \)
   3. Product: \( (fg)(x) = f(x) \cdot g(x) \)
   4. Quotient: \( \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \), provided \( g(x) \neq 0 \).
B. Example

Given \( f(x) = \frac{1}{x} \) and \( g(x) = x - 5 \), find \( f + g \), \( f - g \), \( fg \), \( \frac{f}{g} \). Determine the domain for each function.

\[
D_f : \quad D_g : \\
\]

1. \( (f + g)(x) = \quad D_{f+g} : \quad \)

2. \( (f - g)(x) = \quad D_{f-g} : \quad \)

3. \( (fg)(x) = \quad D_{fg} : \quad \)

4. \( \left( \frac{f}{g} \right)(x) = \quad D_{\frac{f}{g}} : \quad \)

5. \( \left( \frac{g}{f} \right)(x) = \quad D_{\frac{g}{f}} : \quad \)

III. Composite Functions

A. The Composition of a Function

The **composition of function** \( f \) **with** \( g \) is denoted by \( f \circ g \) and is defined by the equation

\[
(f \circ g)(x) = f(g(x)).
\]

The domain of the composite function \( f \circ g \) is the set of all \( x \) such that
1. \( x \) is in the domain of \( g \) and
2. \( g(x) \) is in the domain of \( f \).
B. Examples

1. Given $f(x) = \frac{1}{x}$ and $g(x) = x - 5$, find:

   a. $(f \circ g)(x) = \underline{\hspace{2cm}}$ $(f \circ g)(6) = \underline{\hspace{2cm}}$

   b. $(g \circ f)(x) = \underline{\hspace{2cm}}$ $(g \circ f)(6) = \underline{\hspace{2cm}}$

   c. $(f \circ f)(x) = \underline{\hspace{2cm}}$ $(f \circ f)(6) = \underline{\hspace{2cm}}$

   d. $(g \circ g)(x) = \underline{\hspace{2cm}}$ $(g \circ g)(6) = \underline{\hspace{2cm}}$

2. Find functions $f$ and $g$ such that $h(x) = (f \circ g)(x) = \sqrt{x^2 - 4}$