NET MAPS OF DEGREE 2

Every Thurston map of degree 2 with exactly four postcritical points is a NET map. There are 16 possible dynamic portraits of such maps. The numbering of the portraits is the same as in the enumeration of portraits elsewhere on this web site, but the labellings of the postcritical points by a, b, c, and d is not consistent with the labellings used there.

(1) $\alpha \xrightarrow{2} a \to b \to c \to c, \qquad \beta \xrightarrow{2} d \to b$
(2) $\alpha \xrightarrow{2} a \to b \to c \to b, \qquad \beta \xrightarrow{2} d \to c$
(3) $\alpha \xrightarrow{2} a \to b \to b, \qquad \beta \xrightarrow{2} c \to d \to d$
(4) $\alpha \xrightarrow{2} a \to b \xrightarrow{2} c \to d \to b$
(5) $\alpha \xrightarrow{2} a \to b \to b, \qquad c \xrightarrow{2} d \to c$
(6) $\alpha \xrightarrow{2} a \to b \xrightarrow{2} c \to d \to d$
(7) $\alpha \xrightarrow{2} a \to b \to c \xrightarrow{2} d \to c$
(8) $\alpha \xrightarrow{2} a \to b \to c \to c, \qquad d \xrightarrow{2} d$
(9) $\alpha \xrightarrow{2} a \xrightarrow{2} b \to c \to d \to d$
(10) $\alpha \xrightarrow{2} a \to b \to c \to b, \qquad d \xrightarrow{2} d$
(11) $\alpha \xrightarrow{2} a \xrightarrow{2} b \to c \to d \to c$
(12) $\alpha \xrightarrow{2} a \to b \to c \xrightarrow{2} d \to b$
(13) $a \xrightarrow{2} b \xrightarrow{2} c \to d \to a$
(14) $a \xrightarrow{2} b \to a, \qquad c \xrightarrow{2} d \to c$
(15) $a \xrightarrow{2} b \to c \xrightarrow{2} d \to a$
(16) $a \xrightarrow{2} b \to c \to a, \qquad d \xrightarrow{2} d$

The first three dynamic portraits are portraits of Euclidean Thurston maps. Portraits 8), 10), and 16) are portraits of topological polynomials. In Section 1 we give (in terms of normal forms) the possible rational maps that realize these portraits. In Section 2 for each dynamic portrait we give a subdivision map (for a finite subdivision rule) that realizes the portrait. In Section 3 for each of these subdivision maps we give a wreath recursion for the associated iterated monodromy group. Finally, in Section 4 we give input data for a NET map for each of the 16 portraits.

In general, a given dynamic portrait may be realized by more than one Thurston equivalence class of NET maps. In particular, the rational maps given in Section 1, the subdivision maps gives in Section 2, and the NET maps given by NET map presentations in Section 4 may be in distinct equivalence classes. For more information about the number of equivalence

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classes realizing a given portrait, see Bartholdi-Nekrashevych [1] or Kelsey-Lodge [4].

1. Rational maps

Since the degree is 2, it is straightforward to compute the rational maps realizing a given dynamic portrait. For convenience, we usually normalize so that the first critical value is 0, the second critical value is ∞, and one of the other postcritical points (the image of the first critical value if it isn't already normalized) is 1.

Five of the dynamic portraits, 8), 9), 12), 13), and 16) are only realized by unobstructed maps. This was already known by Bartholdi-Nekrashevych [1] for portrait 16), and for all five it is straightforward to show this by core arc arguments. Here is the idea. We need to show that a Thurston map with this dynamic portrait cannot have a Thurston obstruction. Suppose γ is a simple closed curve that is nontrivial and nonperipheral. The complement of γ in the 2-sphere is a pair of open disks, and each of these disks contains exactly two postcritical points. In each open disk there is an embedded arc that joins the two postcritical points in that disk. Either of these arcs is a *core arc*. The preimage of γ is the boundary of a regular neighborhood of the preimage of either core arc. For each of the five portraits, one can show that the preimage of γ either maps by degree 2 (and so has multiplier 1/2) or isn't in the homotopy class of γ. The arguments don't depend on detailed knowledge of γ, but only on which pairs of postcritical points are in the complementary open disks. Since you can work with either core arc, for each portrait there are only three cases that one has to consider. We will give more detail of the argument in the case of dynamic portrait 8).

(1) $\alpha \xrightarrow{2} a \to b \to c \to c, \qquad \beta \xrightarrow{2} d \to b$

If we set $a = 0$, $b = 1$, and $d = \infty$, then $f(z) = \frac{(z-\alpha)^2}{(z-\beta)^2}$, where either $\alpha = \pm i$ and $\beta = \mp i$ or $\alpha = -1 \pm \sqrt{2}$ and $\beta = -1 \mp \sqrt{2}$.

(2) $\alpha \xrightarrow{2} a \to b \to c \to b, \qquad \beta \xrightarrow{2} d \to c$

If we set $a = 0$, $b = 1$, and $d = \infty$, then $f(z) = \frac{\beta^2}{\alpha^2}\frac{(z-\alpha)^2}{(z-\beta)^2}$, where $\alpha = \frac{1}{2}(-1 \pm \sqrt{7}i)$ and $\beta = \frac{1}{2}(5 \mp \sqrt{7}i)$.

(3) $\alpha \xrightarrow{2} a \to b \to b, \qquad \beta \xrightarrow{2} c \to d \to d$

If we set $a = 0$, $b = 1$, and $c = \infty$, then $f(z) = \frac{\beta^2}{\alpha^2}\frac{(z-\alpha)^2}{(z-\beta)^2}$, where $\alpha = \frac{1}{4}(-1 \pm \sqrt{7}i)$ and $\beta = \frac{1}{16}(5 \mp \sqrt{7}i)$. If we let f' be the function from portrait 2) and let c' be the c from portrait 2), then $f(z) = \frac{1}{c'}f'\left(\frac{1}{z}\right)$.

(4) $\alpha \xrightarrow{2} a \to b \xrightarrow{2} c \to d \to b$

If we set $a = 0$, $b = 1$, and $c = \infty$, we get $f(z) = \frac{1}{\alpha^2}\frac{(z-\alpha)^2}{(z-1)^2}$, where $\alpha = -\frac{1}{2} \pm \frac{1}{2}i$.

(5) $\alpha \xrightarrow{2} a \to b \to b, \qquad c \xrightarrow{2} d \to c$

If we set $a=0$, $b=1$, and $d=\infty$, then $f(z)=\frac{1}{4}\frac{(z+1/2)^2}{(z-1/4)^2}$.

(6) $\alpha \xrightarrow{2} a \to b \xrightarrow{2} c \to d \to d$

If we set $a=0$, $b=1$, and $c=\infty$, we get $f(z)=\frac{1}{\alpha^2}\frac{(z-\alpha)^2}{(z-1)^2}$, where $\alpha=-1\pm i$. If we let f' be the function from portrait 4) and let d' be the d from portrait 4), then $f(z)=\frac{1}{d'}f'\left(\frac{1}{z}\right)$.

(7) $\alpha \xrightarrow{2} a \to b \to c \xrightarrow{2} d \to c$

If we set $a=0$, $b=1$, and $d=\infty$, we get $f(z)=\frac{4(z+2)^2}{(z-4)^2}$. If we let f' be the function from portrait 5) and let c' be the c from portrait 5), then $f(z)=\frac{1}{c'}f'\left(\frac{1}{z}\right)$.

(8) $\alpha \xrightarrow{2} a \to b \to c \to c, \qquad d \xrightarrow{2} d$

If we set $\alpha=0$ and $d=\infty$, then we get the quadratic polynomials $f(z)=z^2+a$, where $a\approx -1.54369$ or $a\approx -0.228155\pm 1.11514i$. One can see easily from a core arc argument that every Thurston map with this portrait is unobstructed. That is, this family is completely unobstructed for the pure modular group. Suppose γ is a nontrivial curve in the complement of the postcritical set that is nonperipheral. We consider a core arc c_γ that is disjoint from γ. It suffices to consider a core arc joining c and d, a core arc joining b and d, and a core arc joining a and d.

Since d maps to itself by degree 2 and b and c map to c by degree 1, the preimage of a core arc c_γ joining c and d will be the union of an arc joining b and d and an arc joining c and d. The boundary of a regular neighborhood of the preimage of c_γ will be peripheral so there can't be a Thurston obstruction with a and b in one complementary component and c and d in the other complementary component. Similar arguments show that there can't be a core arc joining b and d or a core arc joining a and d.

(9) $\alpha \xrightarrow{2} a \xrightarrow{2} b \to c \to d \to d$

If we set $a=0$, $b=\infty$, and $c=1$, we get $f(z)=(z-\alpha)^2/z^2$, where $\alpha\approx 0.456311$ or $\alpha\approx 1.77184\pm 1.11514i$. This family is completely unobstructed for the pure modular group.

(10) $\alpha \xrightarrow{2} a \to b \to c \to b, \qquad d \xrightarrow{2} d$

If we set $\alpha=0$ and $d=\infty$, then we get the quadratic polynomials $f(z)=z^2\pm i$.

(11) $\alpha \xrightarrow{2} a \xrightarrow{2} b \to c \to d \to c$

If we set $a=0$, $b=\infty$, and $c=1$, we get $f(z)=\frac{(z-1/2)^2}{z^2}$.

(12) $\alpha \xrightarrow{2} a \to b \to c \xrightarrow{2} d \to b$

If we set $a=0$, $b=1$, and $d=\infty$, we get $f(z)=\frac{(z-\alpha)^2}{(z+\alpha)^2}$, where $\alpha\approx -3.38298, 0.191488\pm 0.508852i$. This family is completely unobstructed for the pure modular group.

(13) $a \xrightarrow{2} b \xrightarrow{2} c \to d \to a$

If we set $b=0$, $c=\infty$, and $d=1$, then $f(z)=\frac{(z-\frac{1}{2}(3\pm\sqrt{5}))^2}{z^2}$. This family is completely unobstructed for the pure modular group.

(14) $a \xrightarrow{2} b \to a, \qquad c \xrightarrow{2} d \to c$

It is easy to see algebraically that there is no rational map with this dynamic portrait.

(15) $a \xrightarrow{2} b \to c \xrightarrow{2} d \to a$

If we set $b=0$, $c=1$, and $d=\infty$, we get $f(z)=a\frac{(z-a)^2}{(z-1)^2}$, where $a=\frac{1}{2}(-1\pm\sqrt{3}i)$.

(16) $a \xrightarrow{2} b \to c \to a, \qquad d \xrightarrow{2} d$

If we set $a=0$ and $d=\infty$, then we get the quadratic polynomials $f(z)=z^2+b$, where $b\approx -1.75488$ (the airplane) or $b\approx -0.122561\pm 0.744862i$ (the rabbit and the corabbit). From Bartholdi-Nekrashevych [1] or a core arc argument, one can show that this family is completely unobstructed for the pure modular group.

2. Subdivision maps

In this section we give figures for subdivision maps realizing these dynamic portraits. For a figure of a subdivision map $\sigma_{\mathcal{R}}$, the right-hand side shows the 1-skeleton of the subdivision complex $S_{\mathcal{R}}$ (viewing the 2-sphere as the plane compactified by a point at infinity) and the left-hand side shows the 1-skeleton of its subdivision $\mathcal{R}(S_{\mathcal{R}})$. Here a label in black is the label of the point and a label in red is the label of the image point under the subdivision map.

(1) $\alpha \xrightarrow{2} a \to b \to c \to c, \qquad \beta \xrightarrow{2} d \to b$

Figure 1 shows a subdivision map f_1 realizing this portrait.

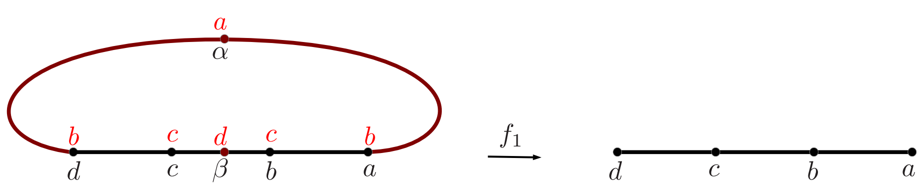

FIGURE 1. The subdivision map f_1.

(2) $\alpha \xrightarrow{2} a \to b \to c \to b, \qquad \beta \xrightarrow{2} d \to c$

Figure 2 shows a subdivision map f_2 realizing this portrait.

(3) $\alpha \xrightarrow{2} a \to b \to b, \qquad \beta \xrightarrow{2} c \to d \to d$

Figure 3 shows a subdivision map f_{3a} realizing this portrait. As was shown in [2], a rational map with this dynamic portrait can not be a subdivision map for a finite subdivision rule whose subdivision complex has 1-skeleton either a tree or a circle. Figure 4 shows an

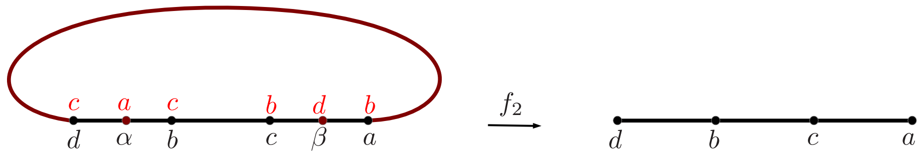

FIGURE 2. The subdivision map f_2.

expanding subdivision map f_{3b} for a finite subdivision rule realizing this dynamic portrait.

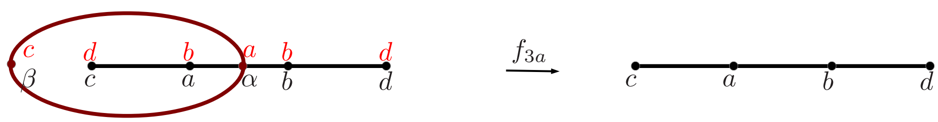

FIGURE 3. The subdivision map f_{3a}.

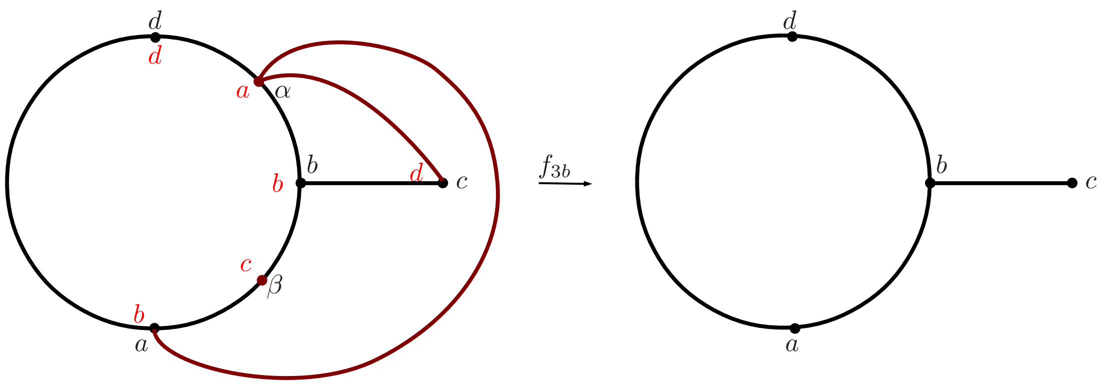

FIGURE 4. The subdivision map f_{3b}.

$$\alpha \xrightarrow{2} a \to b \xrightarrow{2} c \to d \to b \tag{4}$$

$$\alpha \xrightarrow{2} a \to b \to b, \qquad c \xrightarrow{2} d \to c \tag{5}$$

$$\alpha \xrightarrow{2} a \to b \xrightarrow{2} c \to d \to d \tag{6}$$

$$\alpha \xrightarrow{2} a \to b \to c \xrightarrow{2} d \to c \tag{7}$$

$$\alpha \xrightarrow{2} a \to b \to c \to c, \qquad d \xrightarrow{2} d \tag{8}$$

$$\alpha \xrightarrow{2} a \xrightarrow{2} b \to c \to d \to d \tag{9}$$

$$\alpha \xrightarrow{2} a \to b \to c \to b, \qquad d \xrightarrow{2} d \tag{10}$$

$$\alpha \xrightarrow{2} a \xrightarrow{2} b \to c \to d \to c \tag{11}$$

$$\alpha \xrightarrow{2} a \to b \to c \xrightarrow{2} d \to b \tag{12}$$

$$a \xrightarrow{2} b \xrightarrow{2} c \to d \to a \tag{13}$$

$$a \xrightarrow{2} b \to a, \qquad c \xrightarrow{2} d \to c \tag{14}$$

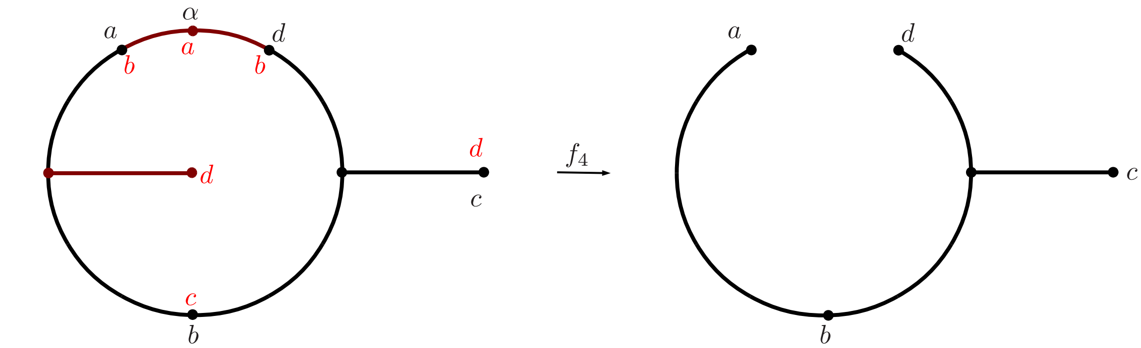

FIGURE 5. The subdivision map f_4.

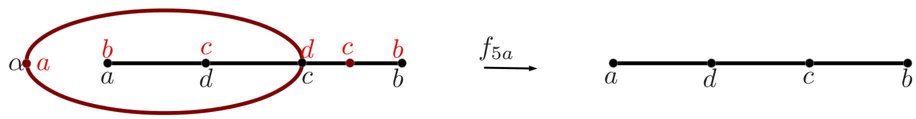

FIGURE 6. The subdivision map f_{5a}.

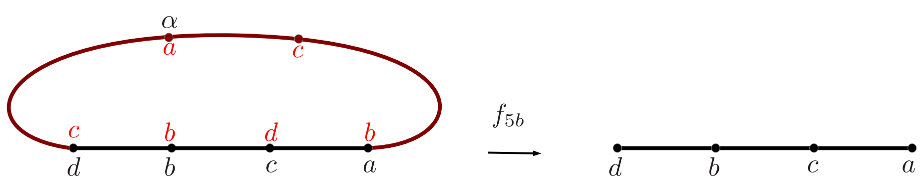

FIGURE 7. The subdivision map f_{5b}.

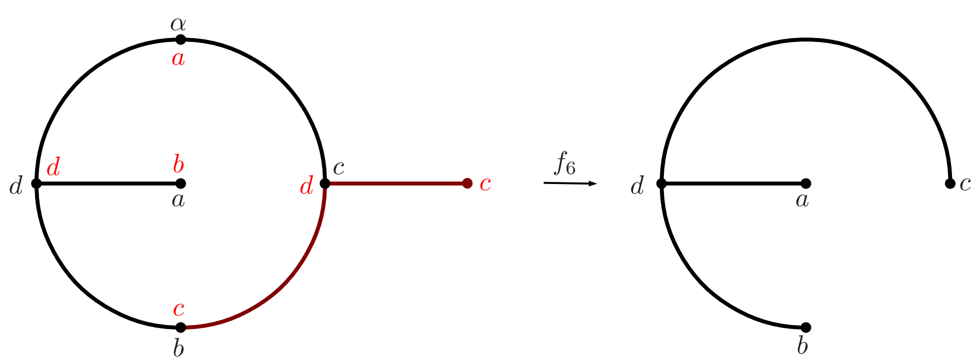

FIGURE 8. The subdivision map f_6.

(15) $a \xrightarrow{2} b \to c \xrightarrow{2} d \to a$

(16) $a \xrightarrow{2} b \to c \to a, \qquad d \xrightarrow{2} d$

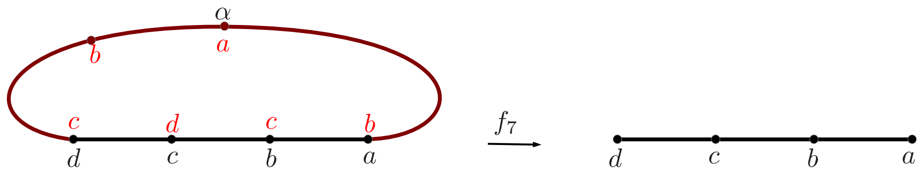

FIGURE 9. The subdivision map f_7.

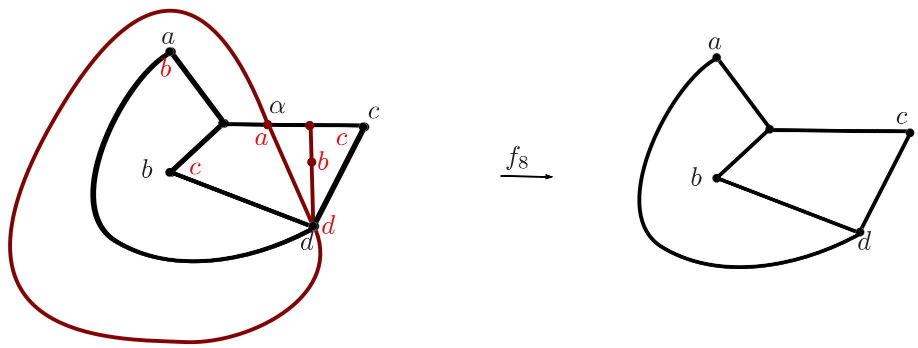

FIGURE 10. The subdivision map f_8.

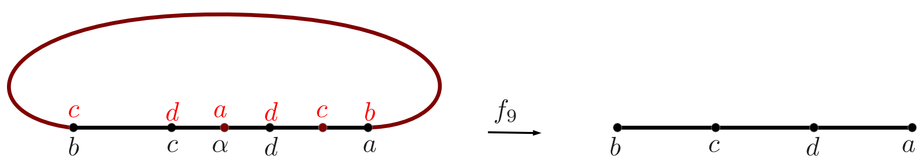

FIGURE 11. The subdivision map f_9.

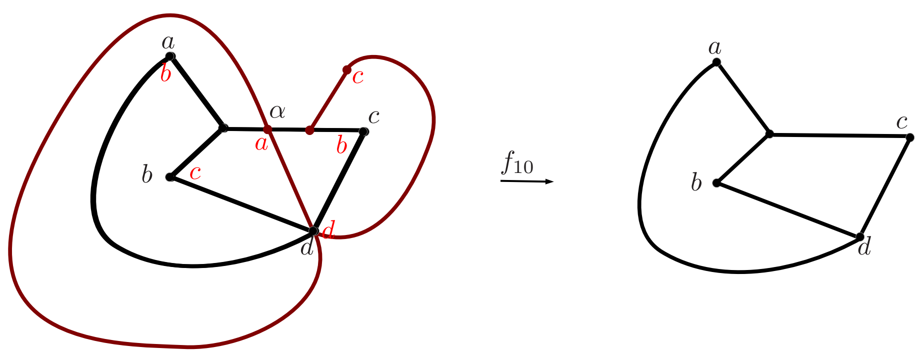

FIGURE 12. The subdivision map f_{10}.

Figure 18 shows the subdivision map f_{16a} (which is equivalent to the airplane) which realizes this dynamic portrait.

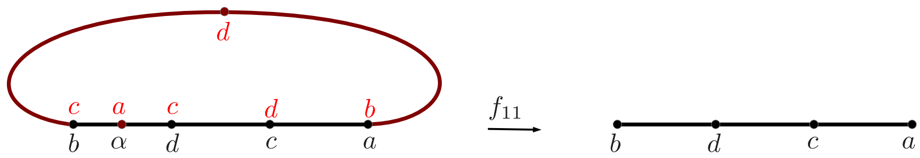

FIGURE 13. The subdivision map f_{11}.

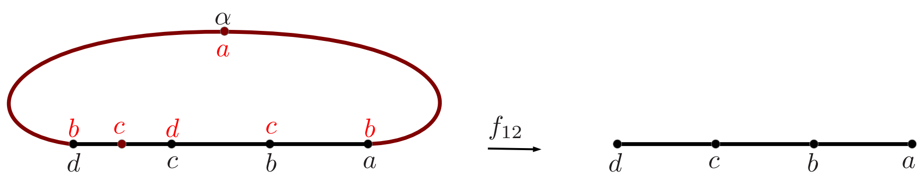

FIGURE 14. The subdivision map f_{12}.

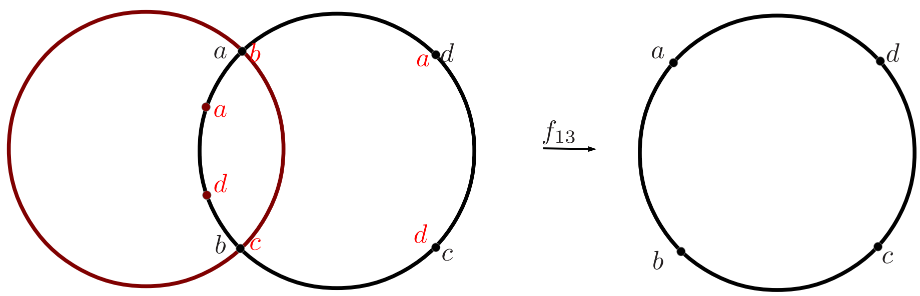

FIGURE 15. The subdivision map f_{13}.

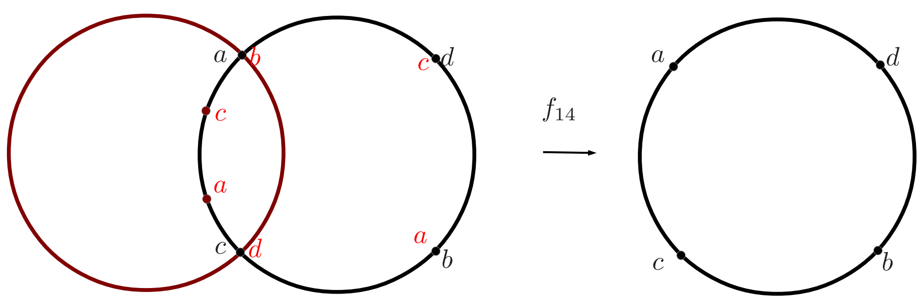

FIGURE 16. The subdivision map f_{14}.

Figure 19 shows the subdivision map f_{16r}, which is equivalent to the rabbit. The 1-skeleton of the subdivision complex $S_{\mathcal{R}}$ shown in Figure 19 comes from a Hubbard tree for the rabbit polynomial together with the rays from a, b, and c to d (which corresponds to ∞).

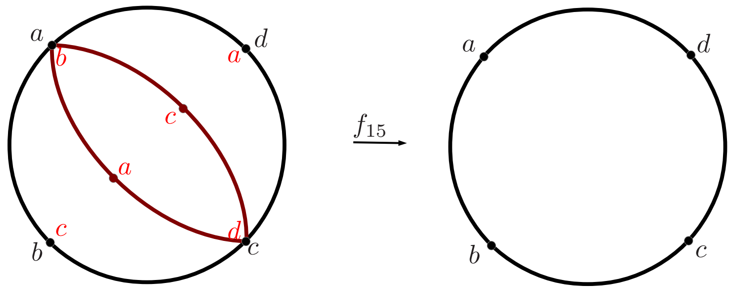

Figure 17. The subdivision map f_{15}.

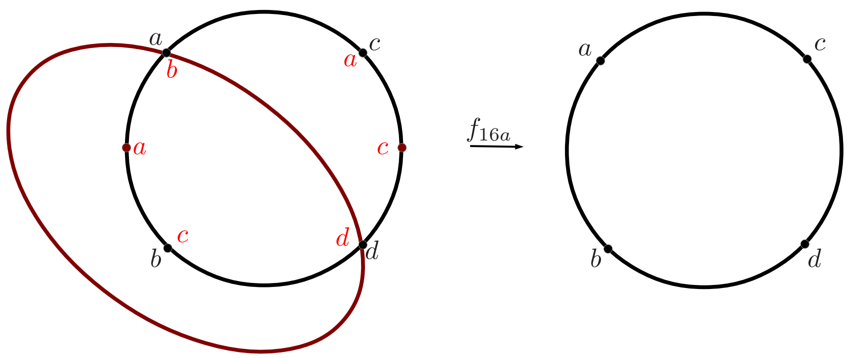

Figure 18. The subdivision map f_{16a}.

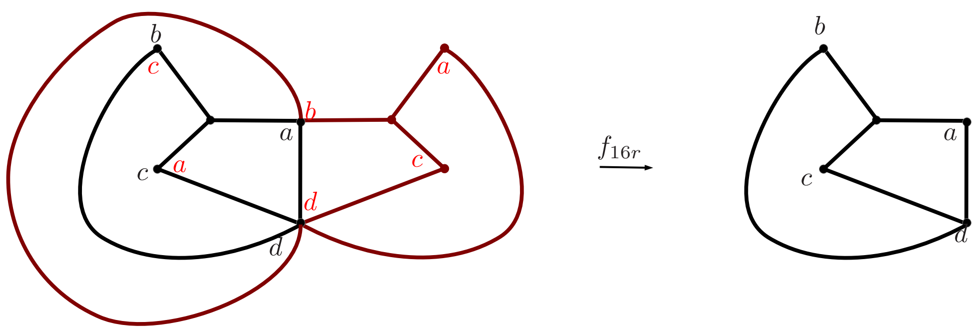

Figure 19. The subdivision map f_{16r}.

3. Wreath recursions

(1) $\alpha \xrightarrow{2} a \to b \to c \to c, \qquad \beta \xrightarrow{2} d \to b$

A wreath recursion for f_1 is as follows:

$$a = \langle 1, 1 \rangle (12), \quad b = \langle a, d \rangle, \quad c = \langle b, dcd^{-1} \rangle, \quad d = \langle ba, dc \rangle (12), \quad dcba = 1$$

Note that in the iterated monodromy group a^2 and d^2 are trivial. Since $b^2 = \langle a^2, d^2 \rangle$, b^2 is also trivial.

(2) $\alpha \xrightarrow{2} a \to b \to c \to b, \qquad \beta \xrightarrow{2} d \to c$

A wreath recursion for f_2 is as follows:

$$a=\langle d^{-1},d\rangle(12),\quad b=\langle a,dbcb^{-1}d^{-1}\rangle,\quad c=\langle d,dbd^{-1}\rangle,\quad d=\langle a,a^{-1}\rangle(12),\quad dbca=1$$

(3) $\alpha \xrightarrow{2} a \to b \to b, \qquad \beta \xrightarrow{2} c \to d \to d$

A wreath recursion for f_{3a} is as follows:

$$a=\langle a^{-1}c^{-1},ca\rangle(12),\quad b=\langle b,cac^{-1}\rangle,\quad c=\langle 1,1\rangle(12),\quad d=\langle d,c\rangle,\quad cabd=1$$

(4) $\alpha \xrightarrow{2} a \to b \xrightarrow{2} c \to d \to b$

A wreath recursion for f_{14} is as follows:

$$a=\langle 1,1\rangle(12),\quad b=\langle a,d\rangle,\quad c=\langle a^{-1},c^{-1}d^{-1}\rangle(12),\quad d=\langle 1,c\rangle,\quad abdc=1$$

(5) $\alpha \xrightarrow{2} a \to b \to b, \qquad c \xrightarrow{2} d \to c$

A wreath recursion for f_{5a} is as follows:

$$a=\langle 1,1\rangle(12),\quad b=\langle a,b\rangle,\quad c=\langle ada^{-1},1\rangle,\quad d=\langle b^{-1},d^{-1}a^{-1}\rangle(12),\quad adcb=1$$

A wreath recursion for f_{5b} is as follows:

$$a=\langle 1,1\rangle(12),\quad b=\langle a,dbd^{-1}\rangle,\quad c=\langle 1,d\rangle,\quad d=\langle b^{-1}d^{-1},a^{-1}\rangle(12),\quad dbca=1$$

(6) $\alpha \xrightarrow{2} a \to b \xrightarrow{2} c \to d \to d$

(7) $\alpha \xrightarrow{2} a \to b \to c \xrightarrow{2} d \to c$

A wreath recursion for f_7 is as follows:

$$a=\langle 1,1\rangle(12),\quad b=\langle a,1\rangle,\quad c=\langle b,d\rangle,\quad d=\langle d^{-1},dc\rangle(12),\quad dcba=1$$

(8) $\alpha \xrightarrow{2} a \to b \to c \to c, \qquad d \xrightarrow{2} d$

A wreath recursion for f_8 is as follows:

$$a=\langle 1,1\rangle(12),\quad b=\langle a,1\rangle,\quad c=\langle b,c\rangle,\quad d=\langle a^{-1}c^{-1},b^{-1}\rangle(12),\quad abdc=1$$

(9) $\alpha \xrightarrow{2} a \xrightarrow{2} b \to c \to d \to d$

A wreath recursion for f_9 is as follows:

$$a=\langle da,bc\rangle(12),\quad b=\langle a,1\rangle(12),\quad c=\langle b,1\rangle,\quad d=\langle c,bcdc^{-1}b^{-1}\rangle,\quad bcda=1$$

(10) $\alpha \xrightarrow{2} a \to b \to c \to b, \qquad d \xrightarrow{2} d$

A wreath recursion f_{10} is as follows:

$$a=\langle 1,1\rangle(12),\quad b=\langle a,c\rangle,\quad c=\langle b,1\rangle,\quad d=\langle a^{-1},c^{-1}b^{-1}\rangle(12),\quad abdc=1$$

(11) $\alpha \xrightarrow{2} a \xrightarrow{2} b \to c \to d \to c$

A wreath recursion for f_{11} is as follows:

$$a=\langle b^{-1},b\rangle(12),\quad b=\langle a,1\rangle(12),\quad c=\langle b,bdb^{-1}\rangle,\quad d=\langle 1,a^{-1}ca\rangle,\quad bdca=1$$

(12) $\alpha \xrightarrow{2} a \to b \to c \xrightarrow{2} d \to b$

A wreath recursion for f_{12} is as follows:

$$a=\langle 1,1\rangle(12),\quad b=\langle a,d\rangle,\quad c=\langle b,1\rangle,\quad d=\langle d^{-1},dc\rangle(12),\quad dcba=1$$

(13) $a \xrightarrow{2} b \xrightarrow{2} c \to d \to a$

A wreath recursion for f_{13} is as follows:

$$a = \langle d, 1\rangle, \quad b = \langle a, 1\rangle(12), \quad c = \langle 1, b\rangle(12), \quad d = \langle c, 1\rangle, \quad abcd = 1$$

(14) $a \xrightarrow{2} b \to a, \qquad c \xrightarrow{2} d \to c$

A wreath recursion for f_{14} is as follows:

$$a = \langle b, 1\rangle, \quad b = \langle a, 1\rangle(12), \quad c = \langle d, 1\rangle, \quad d = \langle 1, c\rangle(12), \quad acbd = 1$$

(15) $a \xrightarrow{2} b \to c \xrightarrow{2} d \to a$

A wreath recursion for f_{15} is as follows:

$$a = \langle d, 1\rangle, \quad b = \langle a, 1\rangle(12), \quad c = \langle 1, b\rangle, \quad d = \langle 1, c\rangle(12), \quad abcd = 1$$

(16) $a \xrightarrow{2} b \to c \to a, \qquad d \xrightarrow{2} d$

A wreath recursion for f_{16a} (the airplane) is:

$$a = \langle c, 1\rangle, \quad b = \langle a, 1\rangle(12), \quad c = \langle 1, b\rangle, \quad d = \langle b^{-1}, bd\rangle(12), \quad abdc = 1$$

A wreath recursion for f_{16r} (the rabbit) is as follows:

$$a = \langle c, 1\rangle, \quad b = \langle 1, a\rangle(12), \quad c = \langle b, 1\rangle, \quad d = \langle cd, c^{-1}\rangle(12), \quad abcd = 1$$

4. Input data for NETmap

The program NETmap takes as input two points λ_1 and λ_2 in the plane, and then six more points, $S_1, \dots, S_6$, giving endpoints for the green line segments. Here are possible inputs for the different possible dynamic portraits, as well as the data in the format $x \mapsto Ax + b$. Except for portraits 10) and 16), the presentations are for the NET maps given in the dynamic portraits lists elsewehere on this site. These NET maps may not be equivalent to the subdivision maps given in Section 2. The presentations given for portraits 10) and 16) are for NET maps equivalent to the maps $z \mapsto z^2 + i$, the rabbit, the corabbit, and the airplane.

(1) $\alpha \xrightarrow{2} a \to b \to c \to c, \qquad \beta \xrightarrow{2} d \to b$

$\lambda_1 = (0, 2)$, $\lambda_2 = (-1, 0)$, $S_1 = (0, 0)$, $S_2 = (0, 2)$, $S_3 = (0, 4)$, $S_4 = (-1, 0)$, $S_5 = (-1, 2)$, $S_6 = (-1, 4)$

$$x \mapsto \begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(2) $\alpha \xrightarrow{2} a \to b \to c \to b, \qquad \beta \xrightarrow{2} d \to c$

$\lambda_1 = (2, 2)$, $\lambda_2 = (0, 1)$, $S_1 = (0, 0)$, $S_2 = (2, 2)$, $S_3 = (4, 4)$, $S_4 = (0, 1)$, $S_5 = (2, 3)$, $S_6 = (4, 5)$

$$x \mapsto \begin{bmatrix} 2 & 0 \\ 2 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

(3) $\alpha \xrightarrow{2} a \to b \to b, \qquad \beta \xrightarrow{2} c \to d \to d$
$\lambda_1 = (2,0)$, $\lambda_2 = (0,1)$, $S_1 = (0,0)$, $S_2 = (2,0)$, $S_3 = (4,0)$, $S_4 = (0,1)$, $S_5 = (2,1)$, $S_6 = (4,1)$

$$x \mapsto \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(4) $\alpha \xrightarrow{2} a \to b \xrightarrow{2} c \to d \to b$
$\lambda_1 = (2,0)$, $\lambda_2 = (0,1)$, $S_1 = (0,0)$, $S_2 = (2,0)$, $S_3 = (4,0)$, $S_4 = (1,0)$, $S_5 = (2,1)$, $S_6 = (4,1)$

$$x \mapsto \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

(5) $\alpha \xrightarrow{2} a \to b \to b, \qquad c \xrightarrow{2} d \to c$
$\lambda_1 = (2,2)$, $\lambda_2 = (0,1)$, $S_1 = (0,0)$, $S_2 = (2,2)$, $S_3 = (4,4)$, $S_4 = (1,1)$, $S_5 = (2,3)$, $S_6 = (4,5)$

$$x \mapsto \begin{bmatrix} 2 & 0 \\ 2 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(6) $\alpha \xrightarrow{2} a \to b \xrightarrow{2} c \to d \to d$
$\lambda_1 = (2,0)$, $\lambda_2 = (0,1)$, $S_1 = (0,0)$, $S_2 = (2,0)$, $S_3 = (4,0)$, $S_4 = (1,0)$, $S_5 = (2,1)$, $S_6 = (4,1)$

$$x \mapsto \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(7) $\alpha \xrightarrow{2} a \to b \to c \xrightarrow{2} d \to c$
$\lambda_1 = (0,2)$, $\lambda_2 = (-1,-1)$, $S_1 = (0,0)$, $S_2 = (0,2)$, $S_3 = (0,4)$, $S_4 = (0,1)$, $S_5 = (-1,1)$, $S_6 = (-1,3)$

$$x \mapsto \begin{bmatrix} 0 & -1 \\ 2 & -1 \end{bmatrix} x + \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

(8) $\alpha \xrightarrow{2} a \to b \to c \to c, \qquad d \xrightarrow{2} d$
$\lambda_1 = (2,2)$, $\lambda_2 = (-1,0)$, $S_1 = (0,0)$, $S_2 = (2,2)$, $S_3 = (4,4)$, $S_4 = (-1,0)$, $S_5 = (1,1)$, $S_6 = (3,4)$

$$x \mapsto \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(9) $\alpha \xrightarrow{2} a \xrightarrow{2} b \to c \to d \to d$
$\lambda_1 = (0,2)$, $\lambda_2 = (-1,0)$, $S_1 = (0,0)$, $S_2 = (0,2)$, $S_3 = (0,4)$, $S_4 = (-1,0)$, $S_5 = (0,1)$, $S_6 = (-1,4)$

$$x \mapsto \begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(10) $\alpha \xrightarrow{2} a \to b \to c \to b, \qquad d \xrightarrow{2} d$
The map $f(z) = z^2 + i$: $\lambda_1 = (2,0)$, $\lambda_2 = (1,1)$, $S_1 = (0,0)$, $S_2 = (2,0)$, $S_3 = (4,0)$, $S_4 = (1,1)$, $S_5 = (1,0)$, $S_6 = (5,1)$

$$x \mapsto \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

(11) $\alpha \xrightarrow{2} a \xrightarrow{2} b \to c \to d \to c$
$\lambda_1 = (2,2)$, $\lambda_2 = (0,1)$, $S_1 = (0,0)$, $S_2 = (2,2)$, $S_3 = (4,4)$, $S_4 = (0,1)$, $S_5 = (1,1)$, $S_6 = (4,5)$

$$x \mapsto \begin{bmatrix} 2 & 0 \\ 2 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

(12) $\alpha \xrightarrow{2} a \to b \to c \xrightarrow{2} d \to b$
$\lambda_1 = (0,2)$, $\lambda_2 = (-1,0)$, $S_1 = (0,0)$, $S_2 = (0,2)$, $S_3 = (0,4)$, $S_4 = (-1,0)$, $S_5 = (0,1)$, $S_6 = (-1,4)$

$$x \mapsto \begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix} x + \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

(13) $a \xrightarrow{2} b \xrightarrow{2} c \to d \to a$
$\lambda_1 = (2,2)$, $\lambda_2 = (-1,0)$, $S_1 = (1,1)$, $S_2 = (2,2)$, $S_3 = (3,3)$, $S_4 = (-1,0)$, $S_5 = (0,1)$, $S_6 = (3,4)$

$$x \mapsto \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(14) $a \xrightarrow{2} b \to a, \qquad c \xrightarrow{2} d \to c$
$\lambda_1 = (2,0)$, $\lambda_2 = (0,1)$, $S_1 = (1,0)$, $S_2 = (2,0)$, $S_3 = (3,0)$, $S_4 = (1,1)$, $S_5 = (2,1)$, $S_6 = (3,1)$

$$x \mapsto \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(15) $a \xrightarrow{2} b \to c \xrightarrow{2} d \to a$
$\lambda_1 = (2,2)$, $\lambda_2 = (0,1)$, $S_1 = (1,1)$, $S_2 = (2,2)$, $S_3 = (3,3)$, $S_4 = (1,2)$, $S_5 = (2,3)$, $S_6 = (3,4)$

$$x \mapsto \begin{bmatrix} 2 & 0 \\ 2 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(16) $a \xrightarrow{2} b \to c \to a, \qquad d \xrightarrow{2} d$
The airplane: $\lambda_1 = (0,-2)$, $\lambda_2 = (1,0)$, $S_1 = (0,-1)$, $S_2 = (0,-2)$, $S_3 = (0,-3)$, $S_4 = (1,0)$, $S_5 = (1,-1)$, $S_6 = (1,-4)$

$$x \mapsto \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The corabbit: $\lambda_1 = (0,1)$, $\lambda_2 = (-2,1)$, $S_1 = (0,0)$, $S_2 = (-1,-3)$, $S_3 = (-2,-4)$, $S_4 = (1,0)$, $S_5 = (0,-1)$, $S_6 = (-1,-4)$

$$x \mapsto \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix} x + \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

The rabbit: $\lambda_1 = (0,-1)$, $\lambda_2 = (2,1)$, $S_1 = (0,0)$, $S_2 = (0,1)$, $S_3 = (0,2)$, $S_4 = (-1,1)$, $S_5 = (-1,2)$, $S_6 = (-2,3)$

$$x \mapsto \begin{bmatrix} 0 & 2 \\ -1 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

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