Introduction

Throughout my experiences with Calculus, I’ve been able to consistently calculate the derivative. This being said, I’ve struggled with particular applications and questions about the derivative which leads me to believe that my understanding is lacking. Wiggins explained how this is possible well when he said,

“understanding is something different than technical prowess; understanding emerges when we are required to reflect upon achievement, to verify or criticize—thus to re-think and re-learn—what we know. Understanding involves questioning …. the assumptions upon which prior learning is based” (1993, p. 8).

As such, as I completed this project, I was hoping to thoroughly investigate my own understanding of the derivative while I learned strategies for teaching the derivative to my future students.

Personal Understanding

When I started this research, I would’ve told a professor, or student, that I knew a lot about the derivative, but certain aspects of it were unclear. For instance, I was not clear how to get from some functions to their derivatives without shortcut rules. I also didn’t feel comfortable with all applications of the derivative. One specific example of this would’ve been related rate problems. In fact, when I completed my original reflection it did not seem that I had mastered very much about the derivative at all. However, instead of investigating a particular type of application, I felt that my understanding would be better bolstered by an investigation of more general derivative-based topics. This would give me more of an overview of topics and help me have a better idea of where my specific deficits were. Overall, my understanding of the derivative could be described as action-view because, though I do not need to think about specific actions to take the derivative, I am unable to fully understand the concepts and meanings behind the derivative and consistently apply them in problems (Oehrtman, 2008, p. 32).

To be specific, there are several student misconceptions which are commonly held amongst calculus students. These are: the derivative at a point gives the function of a derivative, the tangent equation is the derivative function, the derivative at a point is the tangent equation, and, finally, the derivative at a point is the value of the tangent equation at that point (Ubuz, 2001, p. 129). Some of these I was able to immediately discount, but others took a lot more introspection and thought to process.

The first of these misconceptions, the derivative at a point gives the function of a derivative, was one which I was able to discount very quickly, as was the second misconception. However, it took me several minutes to realize that the second two were indeed misconceptions. These can all be broken down as misconceptions due to the fact that the derivative at a point gives the value of the slope of the tangent line to the graph at that particular point. This is an important concept in derivative calculus and thus it is highly important to realize the procedure which connects the two.
A derivative is a rate of change, and the derivative at a particular point is the rate of change of the function at that particular point. Using graphics can help students understand that the derivative at a point gives the value of the slope of the tangent line to the graph at that particular point. This can be shown by using a visual that starts with a secant line and explains that as we make our change in x increasingly smaller, we are getting closer and closer approximations to the rate of change of the derivative at a particular point. Ultimately, as the secant line becomes the tangent line, we are finding the rate of change at the particular point, and since this rate of change is merely the slope of a line, we have shown that the slope of the tangent line to a graph at a particular line is merely the derivative of the function at that particular point, and explained why the procedure of taking slopes of secant lines with increasingly small changes in x’s will eventually give us the derivative at a point.

Student Understanding

The Understanding

Students have limited understanding of derivative calculus, but studies suggest that their understanding is based on limited definitions of the derivative instead of comprehensive definitions of the derivative. For instance, students might think that the derivative at a point is, “the slope of the tangent at a,” or, “the number or the function obtained by applying the usual rules of differentiation, knowing the derivatives of the elementary functions” (Ellison, 1993, p. 34). This is difficult because there are times when certain uses of the derivative are more important to the completion of a problem than are others.

For many students, this will cause problems. Ellison explains that, “mathematicians may be able to grasp all of these conceptions at the same time, choosing from the various possibilities as the mathematical context demands, but what happens to students?” (1993, p. 35) If students have holes in their understanding of the derivative, then they have a limited capacity with which they can work on their derivative problems. This will leave them struggling on simple problems, and unable to fully conquer derivative calculus. This is a problem for students, especially those hoping to continue with calculus work and go into fields which require the use of calculus throughout those students’ work.

The Deficit

Before a student takes calculus they generally go through some sort of sequence of the following courses: trigonometry, algebra, and geometry. These courses are necessary in order to build the mathematical skill set necessary for calculus. In a way, calculus is the culmination of all of these courses for the high school students who take the course. However, Habre discovered the following:

“For instance, in one question students were asked to solve the inequality $3x < 9x + 4$ and to sketch the solution set. Eighty-seven percent of all answers were wrong. When asked to write the equation of the line having slope $-2$ and passing through the point $(-1, 2)$, 71% of all who responded had an incorrect answer. When asked to define the sine, cosine, tangent and cotangent of an angle, 76% did not have a
complete answer. Among other things, students were also asked to complete a table of trigonometric values; the percentage of incorrect answers reached then 87%” (2006, p. 58).

This is extremely alarming as these are all simple skills inherently necessary in a calculus course. How can students understand the concept of creating the equation for a tangent line at a point if they’re unsure how to even write the equation for a line?

Further, students fail to understand the connection between the graphs and the algebra behind derivatives. In a study by Orton, it was discovered that students had, “facility with the symbolic formulas and algorithms of calculus but significant weakness in the graphical representations of the derivative” (Ellison, 1993, p. 37). Most of these students merely had a basic understanding of the derivative and entirely lacked a deep understanding of the derivative concept. Without the ability to connect the algebra to the geometry, there is little chance of students successfully conquering visual representations of derivatives, something which is crucial for a complete understanding.

In Their Words

In one study students were given an experimental calculus course after previously taking a pre-calculus course. The experimental course had a large focus on graphical and analytical understanding of the derivative. Overall, the course was difficult for some students, but extremely enlightening for others. Here are some things students said about the course: “the basic difference is that you understand why things work the way they do,” “It is more elaborate…it concentrates on geometric issues…you have to understand the geometry first, and then comes the algebraic part. This is better,” and finally, “[the course] helps students get ideas from the whole scope” (Habre, 2006, p. 61)

Why This Matters

As students rush through algebra and geometry math courses in order to reach calculus, they are losing the ability to fully comprehend and master simple math concepts. This is especially problematic because the majority of students will never use calculus after graduation. In fact, “many student who take high school calculus have to re-take it in college anyway, because the high school courses don’t cover the same material” (Salzberg, July 17). So, if we’re teaching students advanced concepts in high school, we should at least ensure that they have multiple representations to draw from in college and that we work on understanding the basics before we move on to the more difficult applications of calculus.

Ways to Improve Student Understanding

By the Teacher

There are no substitutes to quality teaching and thorough explanations in the classroom, but teachers can take particular, purposeful actions to help their students better understand the derivative, especially when the derivative is first introduced. One way that teachers can successfully introduce the derivative is suggested by the Calculus Consortium at Harvard
University. They promote a ‘Rule of Four’ in which geometric, numeric, analytic, and verbal explanations of the derivative are given in order to reach students in more ways and create a more complete understanding of the derivative (Kennedy, 1997, p. 6). Specifically, many teachers have found success when students begin to understand the connection between numerical derivatives and graphical derivatives.

One teacher decided to have students ‘discover’ the connection between graphs, derivatives, and power rules on their own. They are given a set of monomials and asked to graph them, and find the connection between the original and the derivative graph. Through their investigation they discover many concepts behind the derivative and are able to fully understand how the power rule comes about. Ultimately, this teacher found that using graphs to introduce this portion of the derivative lead to stronger understanding of the derivative and in less time than he students were before able to do (Novodvorsky, 1998, p. 299).

By the Student

Students can work to master multiple representations of the derivative. As we’ve seen, truly understanding the derivative is something which is not easily done, especially as students take calculus courses earlier and earlier in the school careers. By working to understand multiple representations of the derivative students will learn important critical thinking skills and will become able to apply the concepts of the derivative more frequently and with more accuracy. These representations should include the four suggestions by the Calculus Consortium: geometric, algebraic, numeric, and verbal explanations. Students who are able to master all four of these representations will be better able to understand the derivative and will be able to move past an action-view of derivatives.

Students should also be active participants in their math classes. It is clear that students are not understanding, or at least not retaining, material which is taught to them in their math classes prior to the derivative calculus. Because these topics are so crucial the derivative, it is important that students work to improve their understanding of basic concepts before they ever even reach calculus courses. As students improve their in class behavior, the results will be a positive ripple effect which will make it easier for them to succeed in their later math classes, something which is very important in a subject that depends so much on previously learned materials.

Conclusion

As a result of this project, I have a much stronger understanding of the derivative. I am more aware of various misconceptions which has helped me see where my own comprehension was compromised. By learning about these misunderstandings, I am preparing myself to teach my future students better. Even if I do not end up teaching calculus, I will be able to better prepare my students for a future which will almost definitely require at least one calculus course. Ultimately, I’m really pleased with the way my understanding has improved and I really enjoyed investigating my own understanding in conjunction with research on the understanding of students.
References

Ellison, M. J. (1993). The effect of computer and calculator graphics on students' ability to mentally construct calculus concepts. (volumes I and II)


