1. INTRODUCTION

This study will explore different ways to use trigonometric ideas and specifically how the trigonometric identities are derived. Furthermore, student understanding of these concepts will be discussed. The Pythagorean theorem is often used when working with right triangles, but there is little research on how it is used to derive many other new mathematical ideas. In this study, gaining a better understanding of the trigonometric identities, how they are derived, why they are important, and how students understand them is the ultimate goal. My hope is to delve deeper into the concepts of Pythagorean theorem, right triangles, and the Unit Circle, all of which are useful in deriving trigonometric identities. I want to consider how students understand these ideas and how they may use them in finding trigonometric identities. Additionally, I want to study what students learn and why that may affect their understanding ultimately. The action and process view will also be discussed when reflecting on student understanding.

2. LITERATURE REVIEW

It is essential for students to make connections between mathematical concepts and their associated formulas (Benson & Malm, 2011). Furthermore, teachers should orchestrate the conversation that will help students understand new mathematical ideas (Benson & Malm, 2011). Among the most important topics of trigonometry are right triangles, the Pythagorean theorem, trigonometric functions, and the unit circle. All of these are imperative in developing trigonometric identities. In order to understand the breakdown of trigonometric identities, it is important to understand the concepts that help derive the identities.

2.1 RIGHT TRIANGLES

Grasping ideas of right triangles and exploring new ideas related to them is a step in the direction of being able to derive trigonometric identities. Many teachers entrench trigonometric functions in right triangles, and thus their students are restricted to the use of those functions within the context of right triangles (Moore, LaForest, & Kim, 2012). Students should be able to make the connection that right triangles are related to circles because they should view the hypotenuse as a radius (Moore & LaForest, 2014). Moore & LaForest (2014) stress the importance of understanding this relationship because otherwise, students are unable to give meaning to common trigonometry ratios, such as SOH-CAH-TOA. Using these ratios relates trigonometric functions to angle measures and lengths relative to either the hypotenuse or radius (Moore & LaForest, 2014).
In a study by Cavey & Berenson (2005), one pre-service teacher explained her understanding of right triangles as useful for finding the angles of a triangle. She proceeded to develop a lesson plan based on finding missing parts of a right triangle (Cavey & Berenson, 2005). The furthest she took her lesson was using angle measures to determine similar triangles (Cavey & Berenson, 2005).

2.2 PYTHAGOREAN THEOREM

Galle & Meredith (2014) studied physics students to understand how well they comprehended the ideas of trigonometry. Imperative to understanding the trigonometric identities is the ability to utilize the Pythagorean theorem. They found that these students were quite capable of answering questions pertaining to right triangles (Galle & Meredith, 2014). This includes the uses of sine, cosine, tangent, and the Pythagorean theorem (Galle & Meredith, 2014). Galle & Meredith (2014) go on to advise the use of equations as a tool for improving students’ ability to think conceptually when it comes to trigonometric ideas.

Benson & Malm (2011) discuss how the Pythagorean theorem has many applications. Understanding the theorem itself is the first step in being able to apply it to other mathematical ideas (Benson & Malm, 2011). The article emphasizes the applications of finding an equation for a line, graphing coordinates on the Cartesian plane, using the distance formula, and finding an equation for a circle (Benson & Malm, 2011). Unfortunately, the article does not discuss the Pythagorean theorem’s role in developing the trigonometric identities, nor does it mention how these other applications are also relevant in doing so. Classes that spend more time on the Pythagorean theorem are more likely to become familiar with the relevant concepts and ideas (Lipowsky et al., 2009). Perhaps more time spent covering the Pythagorean theorem would lead to important connections to the trigonometric identities.

2.3 THE UNIT CIRCLE

A wide variety of students do not have a thorough understanding of the unit circle (Moore & LaForest, 2014). Macinnis (2014) combatted this by creating a lesson to make the unit circle real to students. The lesson includes the use of 45-45-90° and 30-60-90° triangles to develop the unit circle (Macinnis, 2014). After doing so, discussion is generated on how the unit circle is used to develop the graphs of the primary trigonometric functions and how those graphs generate identities (Macinnis, 2014). This is the first mention of using a concept in finding the trigonometric identities.

Moore, LaForest, & Kim (2012) emphasize the importance of the unit circle (2012). Both students’ and teachers’ understandings of trigonometric functions lack the ability to make connections to the unit circle (Moore, LaForest, & Kim, 2012). Through studying several students, it was found that the students relied on memorized procedures to complete tasks (Moore, LaForest, & Kim, 2012). Furthermore, any connections discovered were deemed superficial (Moore, LaForest, & Kim, 2012). Some students may not realize that the unit circle does not have be a unit of 1 (Moore, LaForest, & Kim, 2012). Prompting students to think about the unit circle as having differing “units,” the students are then more equipped to use the unit circle, radian measures, and trigonometric functions in novel settings (Moore & LaForest, 2014).
2.4 TRIGONOMETRIC IDENTITIES

In 1989, the National Council of Teachers of Mathematics published a Curriculum and Evaluation Standards for School Mathematics. Even though trigonometry was one of the standards, the only mention of trigonometric identities was to verify them (NCTM, 1989). The Principles and Standards for School Mathematics was updated in 2000, but almost entirely omitted the mention of trigonometry (Brown, 2005). Locally speaking, the Virginia Standards of Learning, last revised in 2009, only mention verifying trigonometric identities. Clearly, there has not been a push forward to fully understand trigonometric identities. This ability to derive the trigonometric identities is the root for being able to fully understand why identities are verified in the first place.

There is very little literature out there that discusses trigonometric identities. Through research, websites were encountered in which explanations were given on how to derive these identities. Due to the lack of research available, perhaps teachers simply aren’t teaching this in their classrooms. The question then becomes whether or not teachers understand it themselves or whether they value it highly enough to teach their students.

3. STUDENT UNDERSTANDING

Research suggests that teachers are lacking the necessary content knowledge in order to provide support to their students’ learning of trigonometry (Moore, LaForest, & Kim, 2012). At some point, definitions must be the foundation upon which other mathematical ideas rest. That being said, however, students may treat equations as definitions rather than new mathematical ideas intended to be discovered and explored. This could likely be due to teachers’ limited content knowledge of trigonometry. One student explained that he thought the hypotenuse was just a side of a triangle; once the connection was made between the hypotenuse and the radius, he was able to make sense of how to find the other lengths (Moore & LaForest, 2014). Without understanding those basic connections, students understanding of a whole concept may prove to be very weak.

A commonly used mnemonic across many trigonometry classrooms is the acronym SOH CAH TOA. This acronym is helpful when deciding which trigonometric function to use with which sides of the triangle. While this is an acceptable definition and even helpful tool for students, they fail to realize its application to more than just finding the numerical lengths of right triangles. Approaching a problem in this way is regarded as an action view (Oehrtman, Carlson, & Thompson, 2008). This view indicates that students can complete step-by-step calculations, but going much further than that is often out of the question (Oehrtman, Carlson, & Thompson, 2008).

Based on the research, it would appear that topics are widely taught as separate entities. Right triangles, the Pythagorean theorem, the Unit Circle, and trigonometric identities may appear to have similar qualities, but are not taught in conjunction. Connections between these trigonometric ideas are not made and thus students have less chance at understanding trigonometric identities. Due to this, students may not be afforded the privilege of being able to think critically and use reasoning. It seems very rare that students are aware that they can use both the Pythagorean theorem and the Unit Circle to derive trigonometric identities. Even when Benson & Malm (2011) conducted their research, which took a deeper look at the Pythagorean theorem, nothing was discussed about the trigonometric identities. Though plotting equations of
circles using the Pythagorean theorem is discussed, no connection is made to the Unit Circle (Benson & Malm, 2011). While these applications of the Pythagorean theorem are exciting, there is still no evidence of how it is used in terms of the trigonometric identities.

4. MY UNDERSTANDING

As a high school student in my trigonometry class, we were expected to utilize memorization. Rather than deriving trigonometric identities within class, we were given the unit circle and formulas to study. We were given timed memorization quizzes and expected to have the trigonometric identities committed to memory. I can distinctly recall cramming the trigonometric identities in my brain before a quiz or test and then furiously writing them down once I was handed my assessment. In Galle & Meredith (2004), using equations as a tool to understand mathematical concepts was encouraged; I, on the other hand, was expected to memorize and apply these ideas to a very specific pool of questions.

A similar technique was used for learning the Pythagorean theorem. I understood that the legs were “a” and “b,” and the hypotenuse was “c.” I understood ideas such as the length of the hypotenuse needed to be greater than each leg of the triangle. What I didn’t understand was the “why.” I didn’t understand how we were able to use the formula $a^2 + b^2 = c$, just that we were expected to do so. Furthermore, I never understood that the hypotenuse of a triangle represented the radius of a circle. Perhaps this is what made it very hard for me to understand and appreciate the trigonometric identities.

In terms of the trigonometric identities, we were given a formula sheet to memorize. Rather than convincing me why I could use them, it was yet another thing to dread. I had no idea what they meant, but was required to know and use them. It wasn’t until this semester that I saw how these were connected. When reviewing trigonometric ideas for the Praxis II exam, a classmate explained how they were derived. Due to the complex and difficult nature of my mathematics classes throughout college, I believed the derivation of the trigonometric identities was just as challenging. I was content just trusting that the identities were accurate. When I realized how the trigonometric identities were derived, I was amazed at the simplicity of it. I immediately wanted to understand the derivation for each identity. The best part is that deriving the trigonometric identities is through the use of right triangles, the Pythagorean theorem, and the Unit Circle, all of which are widely covered in trigonometry classes.

5. CONCLUSION

Students need what Oehrtman, Carlson, & Thompson (2008) refer to as the process view. This approach deals with taking a new idea, manipulating it, and finding results (Oehrtman, Carlson, & Thompson, 2008). This could easily be applied to deriving trigonometric identities. Regardless of the depth of understanding, many students are capable of performing the processes related to right triangles and the Pythagorean theorem. Though connections aren’t always made between those ideas and the Unit Circle, I believe that this is a fairly easy fix. Once these concepts are connected, the opportunities are boundless.
This self-study helped me better understand trigonometry and the amazing connections that exist between trigonometric concepts. Unfortunately, due to the lack of literature and research, I was left with many questions. Based on the research, it appears that the trigonometric identities are widely overlooked. I am curious to understand why trigonometric identities are not derived within the classroom. When research has shown that deeper exploration and connections engage students, why aren’t teachers taking advantage of this? I believe a large factor in this gap falls back on the limited content knowledge of pre-service and in-service teachers. As teachers, we should strive to be masters of our content so as to engage our students. Once we accomplish this, students have the exciting opportunity to explore and understand even the most seemingly difficult mathematical ideas.
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