Supplement: Solids of Revolution

We will look at a few more examples of solids of revolution.

Example: Find the solid generated by revolving the region below the graph of 

\[ f(x) = 0.5 + 0.2 \sin(12x) \]

between \( x = 0.5 \) and \( x = 2.5 \).

The region and the resulting solid are both graphed below:

Here, we can see that slices are perpendicular to the \( x \)-axis, so the radius of each will be given by \( r(x) = \)

We also know that everything must be in terms of \( x \), because we rotated around the \( x \)-axis.

The limits in this case are easy; everything should be in terms of \( x \), and we are specifically restricted to be between

So our integral becomes

This is a challenging integral. To attempt the fundamental theorem, we could try multiplying out the squared term. Then we would still have to find an antiderivative of \( (\sin(12x))^2 \), and we have not learned how to do this.

Instead, we could use a numerical technique like Simpson’s rule. However, because the integrand will oscillate so much, we will have to use a large number of subintervals. You discovered this when you attempted to integrate this very function in Lab 2. However, by using Simpson’s rule with enough subintervals, we determined that the integral comes out to be approximately 1.74 cubic units.
Example 2: Find the solid resulting from rotating \( z = -2x + 1 \), for \( 0 \leq x \leq 0.5 \), about the \( z \)-axis.

In this case, we have rotated about the \( z \)-axis, so everything we do must be in terms of \( z \). We see that the circular slices are perpendicular to the \( z \)-axis, and stretch from the axis to the curve \( z = \ldots \).

We must solve for the radius \( x \) as a function of the height \( z \) in this case;

since \( z = \ldots \), \( x = \ldots \).

(If this seems strange, try drawing in a circular slice and the corresponding radius on the solid and region plotted above. Notice that the radius of each slice is given by an \( x \)-coordinate on the graph, and that the \( x \)-coordinate is a function of the height \( z \).)

We also need limits. Since we rotated about the \( z \)-axis, everything must be in terms of \( z \). The smallest \( z \)-coordinate in our picture is \( z = \ldots \), and the largest is \( z = \ldots \), so our integral for volume is:

Now solve the integral.
*****When working volume problems: *****

1. Start with the picture.
   For solids of revolution, sketch the region involved first, then sketch the solid.
2. Make sure that everything is in terms of the correct variable.
   The axis you rotate about gives the variable to use for a solid of revolution.
   When in doubt, refer to your diagrams.
3. Simplify the integral.
4. Integrate (find the antiderivative)
5. Evaluate using the fundamental theorem.