Answers Homework Lesson 16

Preparation for Lab 4

In Lab 4, we will be estimating the volumes of trunks of trees, which are solids of revolution.

1) There are some unit conversions in this lab. To warm up, make the following conversions:

a) A 2 foot radius corresponds to a diameter in inches of \(48\).

\[
(2)(2)(12) = 48
\]

b) 1,608 in\(^3\) corresponds to how many cubic feet?

\[
1608 \text{ in}^3 \left(\frac{1 \text{ ft}^3}{12 \text{ in}^3}\right) = \frac{1608}{1728} \text{ ft}^3 = 0.93 \text{ ft}^3
\]

2) Now suppose you had the function \((d(h))^2\) for the diameter squared in square-inches for a tree at height \(h\) feet

a) How could you get the radius squared in square inches from \((d(h))^2\)?

If \(d\) is the diameter and \(r\) the radius, then \(\frac{d}{2} = r\), so \(r^2 = \frac{d^2}{4}\) in square inches.

b) How could you get the radius squared in square feet from \((d(h))^2\)?

The radius squared in square inches is \(r^2 = \frac{(d(h))^2}{4}\); so \(r^2\) (in feet\(^2\)) = \(\frac{(d(h))^2}{4 \times 144}\) = \(\frac{(d(h))^2}{576}\)

c) Write down the integral (using \((d(h))^2\)) that would give the volume of the tree in cubic feet between two heights \(h_1\) and \(h_2\) (given in feet).

Since we know the radius squared in square feet, we have:

Volume = \(\int_{h_1}^{h_2} \frac{\pi (d(h))^2}{576} \, dh\) \text{ ft}^3

3) Use what you developed above to solve the following. Suppose that the diameter squared (in square-inches) of a tree at height \(h\) feet is given by the function \((d(h))^2 = 0.36h^2 - 36h + 900\).

Find the volume, in cubic feet, of the tree trunk between \(h = 10\) feet and \(h = 30\) feet.

\[
\int_{10}^{30} \frac{\pi}{576} (0.36h^2 - 36h + 900) \, dh
\]

\[
= \frac{\pi}{576} \left[ \frac{0.36}{3} h^3 - \frac{36}{2} h^2 + 900h \right]_{10}^{30}
\]

\[
= \frac{\pi}{576} \left[ (0.12(30)^3 - 18(30)^2 + 900(30)) - (0.12(10)^3 - 18(10)^2 + 900(10)) \right]
\]

\[
= \frac{\pi}{576} \left[(3240 - 16200 + 27000) - (120 - 1800 + 9000)\right]
\]

\[
= \frac{\pi}{576} (6720) \approx 36.65 \text{ ft}^3
\]