

Lab 2: Left and Right Hand Sums and Trapezoid Rule

Carbon Dioxide in Pond Water

This lab is based on Project 1 for Chapter 5, found on p. 37 in your textbook.

Goals for this lab

- To help you remember what rates of change (derivatives) are and what they mean.
- To use some techniques in Excel (including graphing, setting up left and right hand sums, and setting up the trapezoidal rule) which we will use to analyze data in this class.
- To get practice in interpreting integrals.

Important Note: The online help for this lab (see “Labs” on the 2015 website) includes small samples of what your finished worksheet should look like, as well as brief tutorials on setting up left and right hand sums; make use of as much of this material as you need.

Honor Code Policies

Please read and understand the following before you begin. Honor code policies established by your teacher will govern this lab.

You may complete the lab in a small group of 2-3 people. If you choose to complete the lab in a group, all members must be present (at the same place at the same time), during the completion and write-up of all answers and explanations to all problems in the lab. Each person involved *must* contribute to the work. Each person should in fact spend some time working directly with Excel; it cannot be left to one person to do all of the Excel work in the lab. (In this particular lab, you will have a graph, a left hand sum, a right hand sum, and a trapezoidal rule calculation in Excel. Each person in the group should be at the keyboard typing for at least one of these; other members of the group should still help and be involved even when they are not sitting at the computer.)

Each person who contributed to the lab should get a copy of the completed work to use in answering quiz questions. (We recommend for this reason that you create a complete, neat, and well-organized report, but this is up to you.) No one who did not contribute to the lab should get a copy of this report.

Finally, **each person will take the quiz individually.** When you take the quiz, your work should be your own (based on the lab work you did previously). You are *not* permitted to discuss the questions or answers with anyone other than your instructor under any circumstances. To discuss the questions or answers with anyone else will be considered an Honor Code violation.

Part I: Introduction and Getting the Data

You should read this section individually before your meet with anyone.

The background for this lab is from p. 37 of your textbook, and is reprinted below for your convenience:

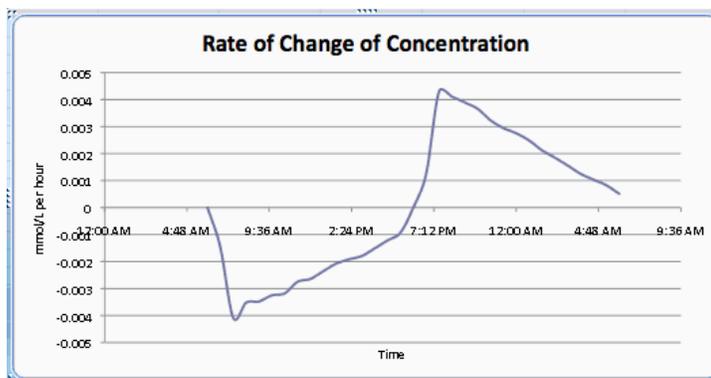
Biological activity in a pond is reflected in the rate at which carbon dioxide, CO_2 , is added to or withdrawn from the water. Plants take CO_2 out of the water during the day for photosynthesis and put CO_2 into the water at night. Animals put CO_2 into the water all the time as they breathe. Biologists are interested in how the net rate at which CO_2 enters a pond varies during the day.... The rate is measured in millimoles (mmol) of CO_2 per liter of water per hour; time is measured in hours past dawn. At dawn, there were 2.600 mmol of CO_2 per liter of water.

You will work with data that gives the *rate of change* of CO₂ concentration as a function of time. (A graph of similar data appears in the textbook, but you will use slightly different data provided for you in an Excel spreadsheet on the 2015 web site. However, you will use the same **initial condition of 2.600 mmol per liter at 6:00 am.**)

Answer the following practice questions for yourself, then check your answers on our web help pages for Lab 2. (These questions are to help make sure you get the interpretation questions in the lab right.)

Practice Question 1: In the sample chart shown below, we show a rate of change of about -0.00328 mmol/L per hour at 9:00 am. What does this mean about the CO₂ concentration at 9:00 am? (Note: Your chart will be different!)

Time	Rate of Change (mmol/L per hour)
6:00 AM	0
6:45 AM	-0.00159219
7:30 AM	-0.00387548
8:15 AM	-0.00340991
9:00 AM	-0.00327837
9:45 AM	-0.00357681



Practice Question 2: Based on the data in the chart above, is the concentration of CO₂ increasing or decreasing at 9:00 am? How can you tell this from your data?

Now get the data you will be working with. It is available on the 2015 Labs web page as the Excel file “Lab2DataF14.xlsx”. Download this file and then open the file in Excel. (Do not open the file in your web browser.)

The worksheet contains two filled-in columns. Under “Time,” you will find the time at which readings were taken. Under “Rate of Change (mmol/L per hour), you will find data on the rate of change of CO₂ concentration at the time the reading was taken.

Part II: Graphing and Analysis of Rates of Change

- Create a graph of the rate of change versus time.
 - Select the appropriate data (all the times and rates of change, but not the titles of these columns).
 - Choose Charts from the choices listed horizontally directly under the tool box.
 - A third menu bar opens. Choose Scatter from the list of choices. (This is the most common graph for this course). Then choose a scatter plot connected with smoothed lines, either with the points showing or not.
- You should click on your chart. With the chart selected:
 - You can move the chart within your excel file.
 - You can add a title and / or label the axes. These are under Chart Layout.
 - Set the title of your chart to “Rate of Change of Concentration.” Label the x and y axes “Time” and “mmol/L per hour,” respectively. A legend is not needed in this plot, so you can click on it and delete it.
 - You can move your chart to a new sheet under Move Chart . . . in the main menu. We won’t do this.

Now answer the following: (You could put the answer in a text box in your Excel worksheet if you wish, or write it down somewhere else.)

Interpretation Question 1: When in the 24 hours shown on your chart is the CO_2 concentration at its *lowest*? How can you tell this from the chart? You may use your data table to help you answer this question. **Remember you are using a rate of change chart to tell you about an amount of change.**

Part III: Left and Right Hand Sums

Now we want to approximate

$$\int_0^{24} (\text{Rate of change of concentration of } CO_2) dt = (\text{Total Change in } CO_2 \text{ concentration over 24 hours})$$

We can approximate this integral with a Right Hand Sum, Left Hand Sum, Trapezoidal Rule and/or Simpson's Rule. In this lab we will consider Right and Left Hand Sums and the Trapezoidal Rule as approximations for this integral.

You will create left and right hand sums using the data on the rate of change of CO_2 concentration. To do this in Excel, you will need to enter a formula. (If you need more help with entering or copying down formulas than is provided here, you can find an on-line "Excel Help Guide" at the 2015 website, on the Labs page.)

For example, suppose we had data for the velocity $v(t)$ (in feet/second) of an object at various times t given in minutes. A table of such data is given at right.

t (min)	$v(t)$ (ft/sec)
2	3
2.5	1
3	2
3.5	2

We know that a left-hand sum can be used to approximate the total distance the object traveled between $t = 2$ minutes and $t = 3.5$ minutes. We need to multiply the first velocity of each subinterval by the **width** of each subinterval to find the distance traveled. In this case, we get

$$\Delta t = 30 \text{ sec.}$$

The width of the interval must match the units on the function $v(t)$, so we must use 30 seconds, rather than 0.5 minutes. Note for example in the first interval, we will get

$$\left(3 \frac{\text{feet}}{\text{sec}}\right)(30 \text{ sec}) = 90 \frac{\text{feet}}{\text{sec}} \text{ sec} = 90 \text{ feet}, \text{ so the units work out.}$$

Since Δt is the same for each subinterval we can factor it out and multiply it by the sum of all the $v(t)$ values except the final value since it is the Left Hand Sum.

We enter the data in Excel

	A	B
1	t	$v(t)$
2	2	3
3	2.5	1
4	3	2
5	3.5	2

This Left Hand Sum can be calculated by multiplying Δt by the sum of 3, 1 and 2 from column B.

Here's how to do this in Excel: Choose an empty cell, B7, for example. Enter the formula “=30*SUM(B2:B4)”.

- An equal sign is needed to tell Excel that you are performing a calculation.
- * is needed to tell Excel to multiply Δt by the sum of the desired numbers of column B.
- SUM(first cell:last cell) will add up all those cells together. We use “SUM(B2 : B4) ” to add All numbers from B2 through B4 in column B.

The result printed in B7 is 180, so we conclude that the object moved about 180 feet.

Note that we do not include row 5, because this corresponds to $t = 3.5$, and that is the *right* hand side of the last subinterval.

A right-hand sum is similar. Examples of both (including some data from this lab) are available under “Examples” in the help pages for Lab 2 on the 2015 web site.

Now it's your turn. For Part III of this lab, complete the following:

1. Set up a left hand sum in Excel for the entire twenty-four hour period. (We have data from 6:00 am to 6:00 am on the following day. Be sure you know what data you are going to use.) Use cell B41, which is marked “Left Hand Sum” on the spreadsheet.

To do this, you will need to determine the width (Δt , in appropriate units) of the subintervals, and enter a formula in cell B41 that multiplies Δt times the sum of the correct values from column B.

Note on scientific notation: When Excel works with very large or very small numbers, it may use scientific notation, which gives a decimal followed by an “E” and a number of places to move the decimal point. It is possible at some point you might see numbers displayed this way. Some examples are:

5.67E-5 is approximately 0.0000567 (decimal moves *left* 5 places for the -5)

1.23E+6 is approximately 1,230,000 (rounded; decimal moves *right* 6 places)

Displaying a number in this manner is common practice for many computer systems; your scientific calculator may do something similar.

2. Set up a right hand sum in Excel using the correct data. Use cell B42, which is marked “Right Hand Sum.” The procedure is similar to the procedure for the left hand sum.
3. Answer the following interpretation question:

Interpretation Question 2: Work out what the units of your left and right hand sum should be. Explain briefly how you determined the units.

Part IV: Trapezoidal Rule

You should be able to complete part IV after you have covered the trapezoidal rule in class.

Recall the Trapezoidal rule:

$$\int_a^b f(x) dx \approx \frac{\Delta x}{2} [f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + \dots + 2f(x_{n-1}) + f(x_n)]$$

where Δx is width of each subinterval and the x_i 's are endpoints of n subintervals.

Now you will set up the trapezoidal rule for the data you have been given. You will need a column for the trapezoidal constants. (For the trapezoidal rule, middle terms are multiplied by 2, but the first and last terms are only multiplied by 1.)

An example of the trapezoidal rule in Excel is shown below, using the same velocity data as we used in our left-hand sum example. We have placed the trapezoidal constants in row C. Column D needs to be column C times column B. The formula in D2 is " $=C2*B2$ " and we complete the column. Check Lab 1 if you don't remember how to complete the column.

For the answer we need to multiply the sum of column D by $\Delta t/2$.

So the formula in D6 is $=\Delta t/2 * \text{SUM}(D2:D5)$.

	A	B	C	D
1	t	$v(t)$	Constants	Trapezoidal
2	2	3	1	3
3	2.5	1	2	2
4	3	2	2	4
5	3.5	2	1	2
6				165

See the online help for this lab if you need other examples of how to set up the columns correctly. You will need to enter the appropriate constants (1s and 2s) in column C, and a formula to multiply the constant column times the function value (column B) in column D. Then the sum of column D must be multiplied by $\Delta t/2$.

When you have finished, check that your trapezoidal answer is indeed the average of the left and right hand sums you calculated. (If it is not the average, something is wrong in one of your three answers!)

Now answer the final interpretation question:

Interpretation Question 3: What is the approximate concentration of CO_2 in the pond water 24 hours from the start of the experiment?

Explain how you got your answer, and precisely *why* the calculations you chose to make will give you the concentration after 24 hours.

Make sure everyone in your group (who participated) gets a copy of the completed lab.