S13 Math 1205 – Test 3 May 1, 2013 NAME:_____ CRN:_____

Use only methods from class. You must show work to receive credit.

1. (11 pts) Let $f(x) = \cos(\pi x) + \sin(\pi x)$.

   (a) (8 pts) Find the linearization $L(x)$ of $f(x)$ centered at $x = 0$.

   $L(x) = 1 + \pi x$

   (b) (3 pts) Would you expect $L(x)$ to give a good approximation to $f(x)$ at $x = 8$? Why or why not?

   No.

2. (14 pts) Let $f(x) = x^2 - a$.

   (a) (2 pts) What value of a will guarantee that $f(x)$ has a root at $\sqrt{2}$?

   $a = 2$

   (b) (12 pts) Using your value for a from above, perform two iterations of Newton's method starting with the initial guess $x_0 = 1$.

   $x_2 = \frac{17}{12}$

3. (18 pts)

   (a) (6 pts) State Rolle's Theorem.

   (b) (12 pts) Let $f(x) = 2x + \cos^2(x)$. Using a theorem from class, show that $f(x)$ cannot have more than one root.

   Hint: Start by assuming $f(x)$ has at least two roots, $x = a$ and $x = b$ $(b > a)$. Also, recall the trigonometric identity $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$.

4. (28 pts) Let $f(x) = 1 - 9x - 6x^2 - x^3$.

   (a) (10 pts) Find the interval(s) where $f(x)$ is decreasing.

   $(-\infty, -3)$ and $(-1, \infty)$

   (b) (4 pts) Find the x-coordinate(s) of the local extrema of $f(x)$, indicating if x is the location of a local minimum or a local maximum.

   local minimum at $x = -3$
   local maximum at $x = -1$.

   (c) (7 pts) Find the interval(s) where $f(x)$ is concave up.

   $(-\infty, -2)$

   (d) (2 pts) Find the x-coordinate(s) of the inflection point(s) of $f(x)$.

   $x = -2$.

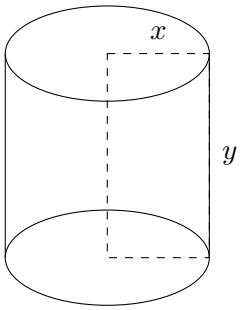
   (e) (5 points) Find the absolute maximum value and absolute minimum value of $f(x)$ on $(-\infty, -1]$. If one or both do not exist, then explain why not.

   Absolute minimum = 1
   Absolute maximum = None

5. (13 pts) Find the third-degree Taylor polynomial centered at $x = 0$ which approximates $f(x) = \frac{1}{2}e^x - \frac{1}{2}e^{-x}$.

   $p_3(x) = x + \frac{1}{6}x^3$

6. (16 pts) A rectangular sheet of perimeter 36 cm and dimensions x cm by y cm is to be revolved about one edge of length y to sweep out a cylinder (see picture below). What values of x and y give the largest cylinder volume? *Hint:* Recall that the volume of a cylinder is $V = \pi r^2 h$, where r is the radius and h is the height.



$x = 12$ cm, $y = 6$ cm

Honor Pledge: I have neither given nor received help on this exam. Signed:_____