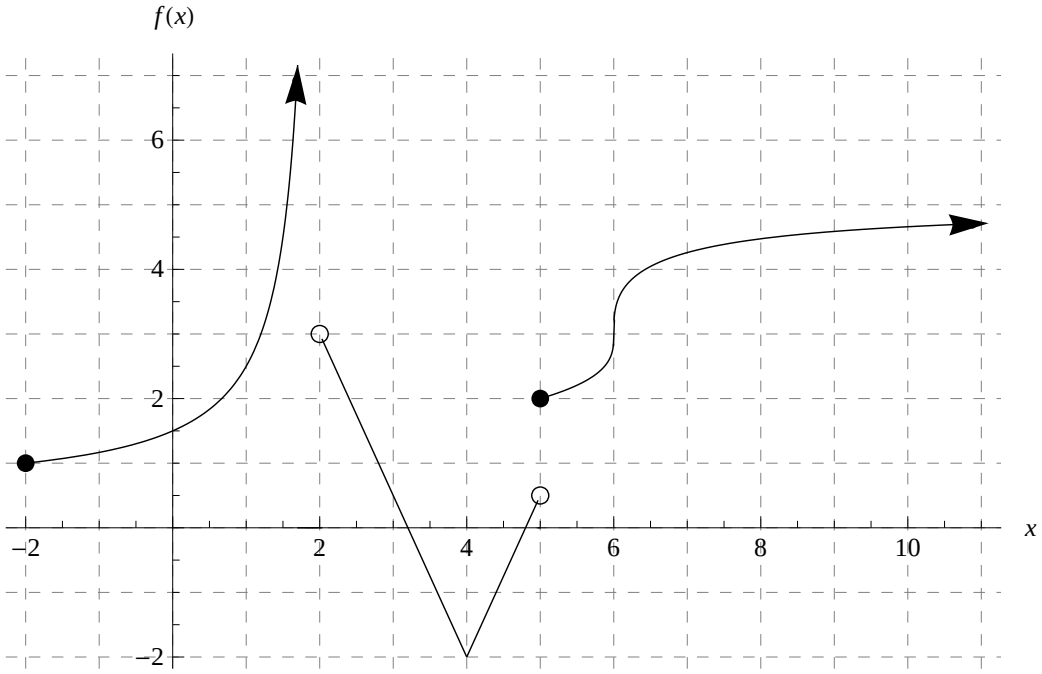
S13 MATH 1205 – Test 1 20 Feb 2013 NAME: _____ CRN:_____

Use only methods from class. You must show work to receive credit.



1. (23 pts) Use the graph above to answer the following. Assume that the only parts of the curve outside this view are as indicated by the arrows.

   (a) (12 pts) Find the following limits using the graph of $f(x)$ above, writing $+\infty$, $-\infty$, and "Does Not Exist" as appropriate.

   i. $\lim_{x\to 5} f(x) =$ DNE

   ii. $\lim_{x\to 2^+} f(x) = 3$

   iii. $\lim_{x\to -2^+} f(x) = 1$

   iv. $\lim_{x\to -2} f(x) =$ DNE

   v. $\lim_{x\to 4} f(x) = -2$

   vi. $\lim_{x\to 6} f(x) = 3$

   (b) (3 pts) List the x-values for each interior point where $f(x)$ is not continuous, if any.

   $x = 2$: function undefined. $x = 5$: jump discontinuity.

   (c) (3 pts) List the equations for all horizontal and vertical asymptotes, if any. Indicate which asymptotes are horizontal and which are vertical.

   horizontal asymptote : $y = 5$, vertical asymptote : $x = 2$

   (d) (5 pts) List the x-values for each interior point where $f(x)$ is not differentiable, if any.

   $x = 2, x = 4, x = 5, x = 6$

2. (22 pts) Evaluate each limit using $+\infty$, $-\infty$ and "Does Not Exist" where appropriate. Use only algebraic methods; tables will not suffice.

   (a) (6 pts) $\lim_{x\to 1^-} \dfrac{x^2+1}{x^2-3x+2}$

   $+\infty$

   (b) (8 pts) $\lim_{x\to 0} \dfrac{\sin^2(4x)}{\sin^2(3x)}$

   $\dfrac{16}{9}$

   (c) (8 pts) $\lim_{x\to 1} \dfrac{\sqrt{x^2+3}-2}{x^2-1}$

   $\dfrac{1}{4}$

3. (7 pts) For $f(x)$ defined below, use the definition of continuity to find the value of a that makes $f(x)$ continuous at $x = 3$.

$$f(x) = \begin{cases} \dfrac{x^2-2x-3}{x^2-x-6}, & x \neq 3 \\ a, & x = 3 \end{cases}$$

   $a = \frac{4}{5}$

4. (10 pts) Using the appropriate limits, give the equation(s) for the horizontal asymptote(s) of $f(x) = \dfrac{2x-4}{|x|}$. Use only algebraic methods; tables will not suffice.

   $y = 2$ and $y = -2$.

5. (12 pts) Let $f(x) = \dfrac{2}{x^2}$ and $c = -1$.

   (a) (2 pts) Find the limit $L = \lim_{x\to c} f(x)$.

   $L = 2$

   (b) (10 pts) Assume $\epsilon = 1$. Find the largest $\delta > 0$ such that for all x, $0 < |x-c| < \delta \implies |f(x) - L| < \epsilon$. Values that might be helpful include: $\sqrt{1/3} \approx 0.6$, $\sqrt{1/2} \approx 0.7$, $\sqrt{2/3} \approx 0.8$, $\sqrt{3/2} \approx 1.2$, $\sqrt{2} \approx 1.4$, $\sqrt{3} \approx 1.7$, $\sqrt{6} \approx 2.5$.

   $\delta = 0.2$

6. (12 pts) Let $f(x) = x^2 + x - 5$.

   (a) (2 pts) In a complete sentence, define what it means for $f(x)$ to have a root at the point $x = c$.

   (b) (10 pts) Using a theorem from class, show that $f(x)$ has two roots in the interval $[-3, 3]$. Justify your work using complete sentences.

7. (14 pts) Let $f(x) = 2x^2 - 1$.

   (a) (8 pts) Use the limit definition of a derivative to find $f'(x)$.

   $f'(x) = 4x$

   (b) (6 pts) Find the equation of the tangent line to $f(x)$ at $x = 2$.

   $y = 8x - 9$

Honor Pledge: I have neither given nor received aid on this exam. Signature: _____