

Name: \_\_\_\_\_

MATH 1205

FINAL

Fall, 2006

ID #: \_\_\_\_\_

CRN: \_\_\_\_\_

(50 pts total)

**Free Response Questions**

**Read the Directions.**

**CALCULATORS MAY BE USED**

**You must SHOW ALL WORK and use methods learned in class to receive full credit.**

1. (8 pts) A manufacturer wishes to minimize the cost of materials to produce 1 liter (=1000 cm<sup>3</sup>) cans (cylindrical). Assuming the thicknesses of the ends and the side are the same, what is the ratio of the height to the diameter of the can that minimizes cost?

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2. (7 pts) Given  $f(x) = \frac{-4(x^2 - x - 2)}{x^2 - 5x + 6}$ . Evaluate the following limits. State  $+\infty$  or  $-\infty$  where appropriate.

a)  $\lim_{x \rightarrow -2} f(x)$

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b)  $\lim_{x \rightarrow 3^-} f(x)$

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3. (12 pts) Find  $\frac{dy}{dx}$  of each of the following functions. Simplify your answers.

a)  $y = \frac{\tan^{-1}(x^2)}{(x^4 + 1)}$

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b)  $y = \ln^4(\sin(3x))$

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c)  $2x^4y^3 - 8x = y$

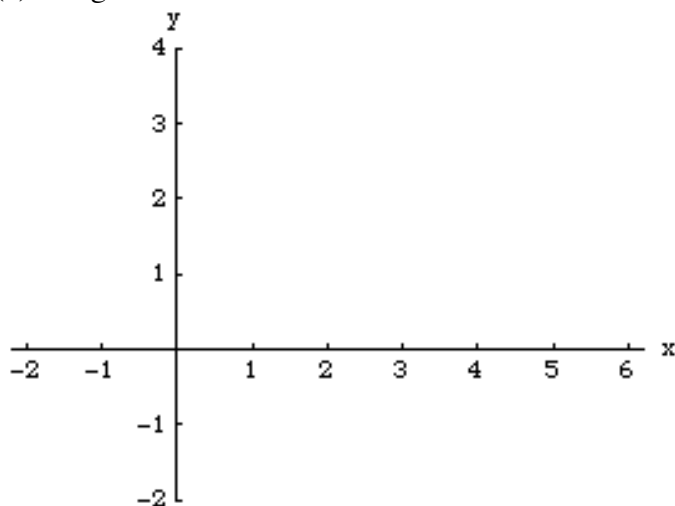
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4. (8 pts) A police cruiser, approaching a right-angled intersection from the north, is chasing a speeding car that has turned the corner and is now moving straight east. When the cruiser is 0.6 mi north of the intersection and the car is 0.8 mi to the east, the police determine with radar that the distance between them and the car is increasing at 20 mph. If the cruiser is moving at 60 mph at the instant of measurement, what is the speed of the car?

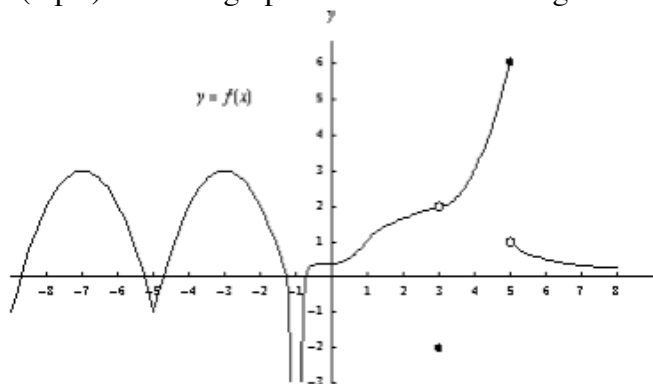
5. (9 pts) Given the following facts about  $f(x)$ ,

- i.)  $f(0) = 1$
- ii.)  $\lim_{x \rightarrow 5^-} f(x) \rightarrow -\infty$
- iii.)  $\lim_{x \rightarrow \infty} f(x) = 2$
- iv.)  $f'(0) = 0$
- v.)  $f'(x)$  does not exist at  $x=2$  and  $x=5$
- vi.)  $f'(x) > 0$  on  $0 < x < 2$  and  $f'(x) < 0$  on  $x < 0, 2 < x < 5$  and  $x > 5$
- vii.)  $f''(x) > 0$  on  $x < 2, 2 < x < 3$  and  $x > 5$  and  $f''(x) < 0$  on  $3 < x < 5$
- viii.)  $f(x)$  is continuous except at  $x=5$ .

- a) What is/are the critical point(s). \_\_\_\_\_
- b) What is the x-coordinate(s) of the local maximum of  $f(x)$  or state that none exist.  
\_\_\_\_\_
- c) What is the x-coordinate(s) of the local minimum of  $f(x)$  or state that none exist.  
\_\_\_\_\_
- d) What is the x-coordinate(s) of the inflection points of  $f(x)$  or state that none exist.  
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- e) Sketch the graph of  $f(x)$  using the information stated above.



6. (6 pts) Use the graph to find the following. If the limit does not exist, give a reason.



- a)  $\lim_{x \rightarrow -5} f(x) =$  \_\_\_\_\_
- b)  $\lim_{x \rightarrow 3} f(x) =$  \_\_\_\_\_
- c)  $\lim_{x \rightarrow 5} f(x) =$  \_\_\_\_\_

BONUS:

Given function  $f$  is continuous on  $[0,4]$  and  $f'(x) = \frac{e^x(x-2)^2}{x^2+4}$  . Is the critical number  $x=2$  a local extreme value? Explain your answer.

Honor Pledge: I have neither given nor received help on this exam.

Signed: \_\_\_\_\_