FORM A

Math 2224 Common Final Exam, Spring 2006

Instructions: Please enter your NAME, your ID NUMBER, the FORM DESIGNATION LETTER and your CRN NUMBER on the op-scan sheet. The index number should be written in the upper right-hand box labeled "Course". Darken the appropriate circles below the ID number and form designation letter. Use a No. 2 pencil; machine grading may ignore faintly marked circles. Mark your answers to the test questions in rows 1–14 of the op-scan sheet. Your score on this part of the test will be the number of correct answers. You have one hour to complete this part of the final exam. Calculators are not allowed.

1. Which one of the following equations describes a hyperboloid of two sheets?

   (1) $x^2 - y^2 + z^2 - 1 = 0$.

   (2) $x^2 - y^2 - z^2 + 1 = 0$.

   (3) $x^2 + 2x - y^2 - z^2 = 0$.

   (4) $x^2 - 2x - y^2 + z^2 + 1 = 0$.

2. At a certain instance, a cylinder with circular cross-section has height 20 cm. which is increasing at the rate of 2 cm/sec., and radius 10 cm. which is decreasing at the rate of 1 cm/sec. Then at this instance, the volume of the cylinder is (recall that the volume of a cylinder with radius r and height h is $\pi r^2 h$)

   (1) Increasing at the rate of 200π cm^3/sec.

   (2) Increasing at the rate of 100π cm^3/sec.

   (3) Decreasing at the rate of 100π cm^3/sec.

   (4) Decreasing at the rate of 200π cm^3/sec.

3. Consider the function $f(x,y) = x^2y^2 + 2x + 2y$. Which one of the following statements is true?

   (1) f has no critical points in the (x,y)-plane.

   (2) There is a point in the (x,y)-plane where f is a local maximum.

   (3) There is a point in the (x,y)-plane where f is a local minimum.

   (4) There is a point in the (x,y)-plane where f is a saddle point.

4. Let D denote the closed disk of radius 1 with center the origin, that is $\{(x,y) \mid x^2+y^2 \leq 1\}$. Then the maximum value of the function x^2y on D is

   (1) $\dfrac{1}{3\sqrt{3}}$

   (2) $\dfrac{2}{3\sqrt{3}}$

   (3) $\dfrac{1}{\sqrt{3}}$

   (4) $\dfrac{2}{\sqrt{3}}$

5. The value of $\displaystyle\int_0^2 \int_{y/2}^1 e^{2x^2}\,dxdy$ is

   (1) $(e^2-1)/2$

   (2) $(e^2-1)/4$

   (3) $(e^2+1)/4$

   (4) $(e^2-2)/4$

6. The value of $\displaystyle\int_0^2 \int_0^{\sqrt{4-x^2}} \sqrt{x^2+y^2}\,dydx$ is

   (1) $\pi/2$

   (2) π

   (3) $4\pi/3$

   (4) 2π

7. The equation of the tangent plane to the surface $x^2+2y^2+3z^2=6$ at the point $(2,1,0)$ is

   (1) $x+2y+3z=3$

   (2) $x+2y+3z=6$

   (3) $x+y=3$

   (4) $x+y=6$

8. The infinite series $\sum_{n=1}^{\infty} \tan^{-1}(n) - \tan^{-1}(n-1)$ (where $-\pi/2 < \tan^{-1} n < \pi/2$) has sum

   (1) $-\pi/4$

   (2) 0

   (3) $\pi/4$

   (4) $\pi/2$

9. The first three nonzero terms of the Taylor series for $f(x) = \cos(\pi x)$ expanded about the point $x = 1/3$ are:

   (1) $1 - \pi(x-1/3)/2 + \pi^2(x-1/3)^2/4$

   (2) $1 - \pi(x-1/3)/2 - \pi^2(x-1/3)^2/4$

   (3) $1/2 - \pi(x-1/3)/2 + \pi^2(x-1/3)^2/4$

   (4) $1/2 - \sqrt{3}\pi(x-1/3)/2 - \pi^2(x-1/3)^2/4$

10. Let T be the tetrahedron with vertices $(0,0,0)$, $(2,0,0)$, $(1,1,0)$ and $(0,0,1)$. Then the volume of T is given by

    (1) $\int_0^1 \int_y^{2-y} \int_0^{2-x-y} dz\,dx\,dy$

    (2) $\int_0^1 \int_y^{1-y} \int_0^{1-x/2-y/2} dz\,dx\,dy$

    (3) $\int_0^1 \int_x^{1-x} \int_0^{1-x/2-y/2} dz\,dy\,dx$

    (4) $\int_0^1 \int_0^{1-z} \int_y^{2-y-2z} dx\,dy\,dz$

11. The integral $\int_0^{2\pi} \int_0^{\pi/4} \int_0^1 \rho\, d\rho\, d\phi\, d\theta$ in cylindrical coordinates is

    (1) $\int_0^{2\pi} \int_0^1 \int_r^{\sqrt{1-r^2}} r\, dz\,dr\,d\theta$

    (2) $\int_0^{2\pi} \int_0^{1/\sqrt{2}} \int_r^{\sqrt{1-r^2}} dz\,dr\,d\theta$

    (3) $\int_0^{2\pi} \int_0^1 \int_{-r}^{\sqrt{1-r^2}} r\, dz\,dr\,d\theta$

    (4) $\int_0^{2\pi} \int_0^1 \int_{-r}^{\sqrt{1-r^2}} dz\,dr\,d\theta$

12. Let $a_n = \dfrac{1}{100} + \dfrac{100}{n}$ and let $b_n = (-1)^n a_n$. Which one of the following statements is true?

    (1) The sequence $\{a_n\}$ is convergent and the series $\sum b_n$ is convergent.

    (2) The sequence $\{a_n\}$ is convergent and the series $\sum b_n$ is divergent.

    (3) The sequence $\{a_n\}$ is divergent and the series $\sum b_n$ is divergent.

    (4) The sequence $\{a_n\}$ is divergent and the series $\sum b_n$ is convergent.

13. The sum of the series $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n 2^n}{3^n}$ is

    (1) 4/15

    (2) 4/3

    (3) 2/5

    (4) 2

14. The radius of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 2^n}{n!}(x-1)^n$ is

    (1) 1/2

    (2) 1

    (3) 2

    (4) ∞