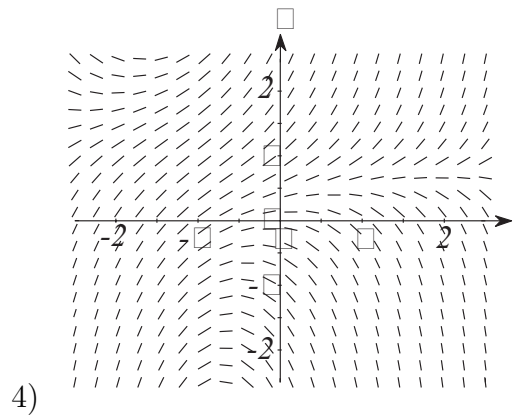
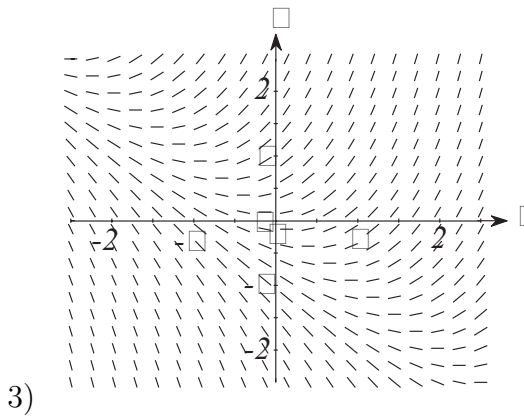
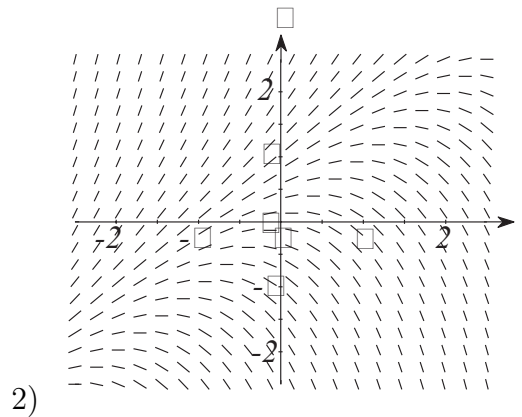
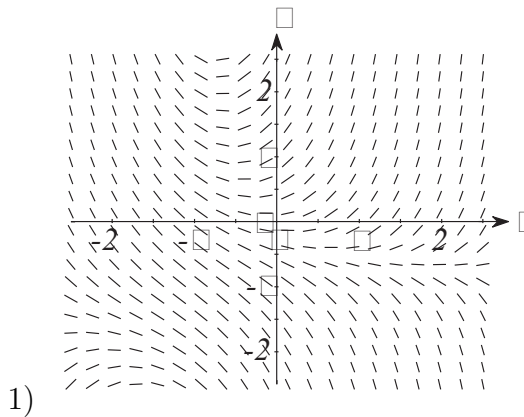


FORM A

Instructions: Please enter your NAME, your ID NUMBER, the FORM DESIGNATION LETTER and your CRN NUMBER on the op-scan sheet. The index number should be written in the upper right-hand box labeled "Course". Darken the appropriate circles below the ID number and form designation letter. Use a No. 2 pencil; machine grading may ignore faintly marked circles. Mark your answers to the test questions in rows 1- 12 of the op-scan sheet. Your score on this part of the test will be the number of correct answers. You have one hour to complete this part of the final exam.

[1] Which direction field corresponds to the differential equation $y' = y - t$?



[2] What is the largest open interval containing $t = 2$ in which the solution of the initial value problem $(t^2 - 9)y' + (t - 1)y = 3 \ln(|t|)$, $y(2) = -1$, is certain to exist?

- 1) $(0, \infty)$ 2) $(-3, 3)$ 3) $(0, 3)$ 4) $(1, 3)$

[3] Let $y(t)$ be the solution of the initial value problem $y' = y - t^2$, $y(1) = 2$. Using Euler's method with step size $h = 1/2$ to approximate $y(2)$, one obtains

- 1) 2 2) $\frac{5}{2}$ 3) $\frac{11}{4}$ 4) $\frac{21}{8}$

[4] Let $y(t)$ be the solution of the initial value problem $y' - 2y = t$, $y(0) = 1$. Then $y(1)$ equals

- 1) $-\frac{1}{4} + e^2$ 2) $-\frac{3}{4} + \frac{5}{4}e^2$ 3) $\frac{1}{4} + \frac{1}{4}e^2$ 4) $\frac{1}{2}$

[5] Find a constant b so that $\mathbf{y}(t) = e^{2t} \begin{bmatrix} 1 \\ 4 \\ b \end{bmatrix}$ is a solution of $\mathbf{y}' = \begin{bmatrix} 4 & 1 & 3 \\ 2 & 3 & 3 \\ -2 & -1 & -1 \end{bmatrix} \mathbf{y}$.

- 1) -4 2) -2 3) 2 4) 4

[6] Rewrite the differential equation $y''' - ty' - 6y = \cos(t)$ as a system $\mathbf{Y}' = P(t)\mathbf{Y} + \mathbf{G}(t)$ of first order linear equations.

- 1) $Y' = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & t & 0 \end{bmatrix} Y + \begin{bmatrix} 0 \\ 0 \\ \cos(t) \end{bmatrix}$ 2) $Y' = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ t & 6 & 0 \end{bmatrix} Y + \begin{bmatrix} 0 \\ 0 \\ \cos(t) \end{bmatrix}$
- 3) $Y' = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & t & 6 \end{bmatrix} Y + \begin{bmatrix} 0 \\ 0 \\ \cos(t) \end{bmatrix}$ 4) $Y' = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 6 & t \end{bmatrix} Y + \begin{bmatrix} 0 \\ 0 \\ \cos(t) \end{bmatrix}$

[7] Let $y(t)$ be the solution of $y' - \sin(t)y^2 = 0$, $y(0) = \frac{1}{3}$. Then $y(\pi)$ equals:

- 1) $-\frac{5}{3}$ 2) $\frac{1}{5}$ 3) $\frac{1}{2}$ 4) 1

[8] The general solution of $\mathbf{y}'(t) = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix} \mathbf{y}$ is

$$1) \mathbf{y}(t) = \begin{bmatrix} c_1 e^{2t} + c_2 e^{3t} \\ c_1 e^{2t} + 2c_2 e^{3t} \end{bmatrix} \quad 2) \mathbf{y}(t) = \begin{bmatrix} c_1 e^{2t} + c_2 e^{3t} \\ -c_1 e^{2t} + 2c_2 e^{3t} \end{bmatrix}$$

$$3) \mathbf{y}(t) = \begin{bmatrix} c_1 e^{2t} + c_2 e^{3t} \\ c_1 e^{2t} - 2c_2 e^{3t} \end{bmatrix} \quad 4) \mathbf{y}(t) = \begin{bmatrix} c_1 e^{2t} + c_2 e^{3t} \\ -c_1 e^{2t} - 2c_2 e^{3t} \end{bmatrix}$$

[9] According to the method of undetermined coefficients, the correct form of a particular solution of $y'' + y' - 2y = \cos(2t) + te^{-2t}$ is

$$1) At \cos(2t) + Bt \sin(2t) + (Ct + D)e^{-2t} \quad 2) A \cos(2t) + B \sin(2t) + (Ct + D)e^{-2t}$$
$$3) At \cos(2t) + Bt \sin(2t) + (Ct^2 + Dt)e^{-2t} \quad 4) A \cos(2t) + B \sin(2t) + (Ct^2 + Dt)e^{-2t}$$

[10] Find a constant α so that the solution $y(t)$ of the initial-value problem $y'' - 2y' - 8y = 0$, $y(0) = 2$, $y'(0) = \alpha$, approaches 0 as $t \rightarrow \infty$.

$$1) -4 \quad 2) -2 \quad 3) 2 \quad 4) 4$$

[11] Suppose that the general solution of the homogeneous, second order equation

$$y'' + p(t)y' + q(t)y = 0, \quad 0 < t < \infty$$

is $y(t) = c_1 \frac{1}{t} + c_2 t^3$. A particular solution of $y'' + p(t)y' + q(t)y = 1$ is

$$1) -\frac{t^2}{3} \quad 2) \frac{t}{2} - \frac{1}{3} \quad 3) -\frac{t^2}{2} \quad 4) \frac{t^2}{6}$$

[12] A 100 gallon tank originally contains 20 gallons of water and 5 lb of salt. Then water containing $1/2$ lb salt per gallon is poured into the tank at the rate of 2 gallons per minute, and the well-stirred mixture leaves at a rate of 1 gallons per minute. Find the amount of salt after 5 minutes.

$$1) 7 \text{ lbs} \quad 2) 7\frac{1}{2} \text{ lbs} \quad 3) 8\frac{1}{2} \text{ lbs} \quad 4) 10 \text{ lbs}$$