[1] Which direction field corresponds to the differential equation $y' = y - t$?
[2] What is the largest open interval containing \( t = 2 \) in which the solution of the initial value problem \((t^2 - 9)y' + (t - 1)y = 3 \ln(|t|), \ y(2) = -1\), is certain to exist?

1) \((0, \infty)\) 2) \((-3, 3)\) 3) \((0, 3)\) 4) \((1, 3)\)

[3] Let \( y(t) \) be the solution of the initial value problem \( y' = y - t^2, \ y(1) = 2\). Using Euler’s method with step size \( h = 1/2 \) to approximate \( y(2) \), one obtains

1) 2 2) \(\frac{5}{2}\) 3) \(\frac{11}{4}\) 4) \(\frac{21}{8}\)

[4] Let \( y(t) \) be the solution of the initial value problem \( y' - 2y = t, \ y(0) = 1\). Then \( y(1) \) equals

1) \(-\frac{1}{4} + e^2\) 2) \(-\frac{3}{4} + \frac{5}{4} e^2\) 3) \(\frac{1}{4} + \frac{1}{4} e^2\) 4) \(\frac{1}{2}\)

[5] Find a constant \( b \) so that \( y(t) = e^{2t} \begin{bmatrix} 1 \\ 4 \\ b \end{bmatrix} \) is a solution of \( \begin{bmatrix} 4 & 1 & 3 \\ 2 & 3 & 3 \\ -2 & -1 & -1 \end{bmatrix} \).

1) \(-4\) 2) \(-2\) 3) 2 4) 4

[6] Rewrite the differential equation \( y''' - ty' - 6y = \cos(t) \) as a system \( \mathbf{Y}' = P(t) \mathbf{Y} + \mathbf{G}(t) \) of first order linear equations.

1) \( \mathbf{Y}' = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & t & 0 \end{bmatrix} \mathbf{Y} + \begin{bmatrix} 0 \\ 0 \\ \cos(t) \end{bmatrix} \) 2) \( \mathbf{Y}' = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ t & 6 & 0 \end{bmatrix} \mathbf{Y} + \begin{bmatrix} 0 \\ 0 \\ \cos(t) \end{bmatrix} \)

3) \( \mathbf{Y}' = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & t & 6 \end{bmatrix} \mathbf{Y} + \begin{bmatrix} 0 \\ 0 \\ \cos(t) \end{bmatrix} \) 4) \( \mathbf{Y}' = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 6 & t \end{bmatrix} \mathbf{Y} + \begin{bmatrix} 0 \\ 0 \\ \cos(t) \end{bmatrix} \)

[7] Let \( y(t) \) be the solution of \( y' - \sin(t)y^2 = 0, \ y(0) = \frac{1}{3} \). Then \( y(\pi) \) equals:

1) \(-\frac{5}{3}\) 2) \(\frac{1}{5}\) 3) \(\frac{1}{2}\) 4) 1
[8] The general solution of $y'(t) = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix} y$ is

1) $y(t) = \begin{bmatrix} c_1 e^{2t} + c_2 e^{3t} \\ c_1 e^{2t} + 2c_2 e^{3t} \end{bmatrix}$  

2) $y(t) = \begin{bmatrix} c_1 e^{2t} + c_2 e^{3t} \\ -c_1 e^{2t} + 2c_2 e^{3t} \end{bmatrix}$

3) $y(t) = \begin{bmatrix} c_1 e^{2t} + c_2 e^{3t} \\ c_1 e^{2t} - 2c_2 e^{3t} \end{bmatrix}$

4) $y(t) = \begin{bmatrix} c_1 e^{2t} + c_2 e^{3t} \\ -c_1 e^{2t} - 2c_2 e^{3t} \end{bmatrix}$

[9] According to the method of undetermined coefficients, the correct form of a particular solution of $y'' + y' - 2y = \cos(2t) + te^{-2t}$ is

1) $A \cos(2t) + Bt \sin(2t) + (Ct^2 + D) e^{-2t}$

2) $A \cos(2t) + B \sin(2t) + (Ct^2 + D) e^{-2t}$

3) $A \cos(2t) + Bt \sin(2t) + (Ct^2 + Dt) e^{-2t}$

4) $A \cos(2t) + B \sin(2t) + (Ct^2 + Dt) e^{-2t}$

[10] Find a constant $\alpha$ so that the solution $y(t)$ of the initial-value problem $y'' - 2y' - 8y = 0$, $y(0) = 2$, $y'(0) = \alpha$, approaches 0 as $t \to \infty$.

1) $-4$  

2) $-2$

3) $2$

4) $4$

[11] Suppose that the general solution of the homogeneous, second order equation

$y'' + p(t)y' + q(t)y = 0$, $0 < t < \infty$

is $y(t) = c_1 \frac{1}{t} + c_2 t^3$. A particular solution of $y'' + p(t)y' + q(t)y = 1$ is

1) $-\frac{t^2}{3}$

2) $\frac{t}{2} - \frac{1}{3}$

3) $-\frac{t^2}{2}$

4) $\frac{t^2}{6}$

[12] A 100 gallon tank originally contains 20 gallons of water and 5 lb of salt. Then water containing $\frac{1}{2}$ lb salt per gallon is poured into the tank at the rate of 2 gallons per minute, and the well-stirred mixture leaves at a rate of 1 gallons per minute. Find the amount of salt after 5 minutes.

1) 7 lbs  

2) $\frac{7}{2}$ lbs

3) $\frac{8}{2}$ lbs

4) 10 lbs