1. A hot air balloon is launched vertically from the ground level at time $t = 0$. Assume that the initial velocity was zero and the motion is strictly vertical. Due to cooling, the vertical acceleration of the balloon is changing over time and can be modeled by

$$a(t) = -32 + \frac{64}{(t + 1)^3}$$

Determine the position and the velocity of the balloon at the time $t = 1$.

(A) $p = 32; \ v = 0$  \quad (B) $p = 16; \ v = -16$

(C) $p = 0; \ v = -8$  \quad (D) $p = 0; \ v = -40$
2. A sloped roof of a building accumulates snow during the winter. The snow thickness \( h(x) \) depends on the slope of the roof \( m \geq 0 \) and can be expressed with the following formula:

\[
h(x) = \frac{1}{m+1} (1 - x^2) \text{ measured in feet, } -1 \leq x \leq 1
\]

where \( x \) is the horizontal distance from the center of the building to the side walls. Determine the value of \( m \) that will ensure that average thickness of the snow is \( \frac{1}{2} \) ft.

(A) \( m = \frac{5}{3} \)  
(B) \( m = \frac{1}{3} \)  
(C) \( m = 1 \)  
(D) \( m = 0 \)

3. Evaluate \( \int -\frac{1}{x^2} \ln(x^2 + 1) \, dx \)

(A) \( \frac{1}{x} \ln(x^2 + 1) - 2 \tan^{-1} x + C \)  
(B) \( \frac{1}{x} \ln(x^2 + 1) - 2 \ln(x^2 + 1) + C \)

(C) \( \frac{1}{x} \ln(x^2 + 1) + C \)  
(D) \( \frac{1}{x} \ln(x^2 + 1) - \ln |x| + \frac{1}{2} \ln(x^2 + 1) + C \)

4. Suppose a thin plate of density \( \delta = \frac{1}{8} \) is bounded by the lines \( y = ax, \ y = 0, \) and \( x = 2, \) where \( a \) is a constant. If the thin plate’s moment about the \( y \)-axis is \( \frac{1}{4}, \) what is the value of \( a? \)

(A) \( \frac{3}{4} \)  
(B) \( \frac{3}{32} \)  
(C) \( 1 \)  
(D) \( \frac{3}{2} \)

5. After an appropriate trig substitution, the integral \( \int \frac{x^2}{\sqrt{1-9x^2}} \, dx \) becomes

(A) \( \int 27 \sin^2 \theta \, d\theta \)  
(B) \( \int \frac{1}{27} \sin^2 \theta \, d\theta \)

(C) \( \int \frac{9 \sin^2 \theta}{\cos \theta} \, d\theta \)  
(D) \( \int \frac{\sin^2 \theta}{9 \cos \theta} \, d\theta \)

6. Evaluate \( \int \frac{x^2 + x + 2}{x + 4} \, dx \)

(A) \( \frac{1}{2} x^2 - 10 \ln |x| + C \)  
(B) \( \frac{1}{2} x^2 + 14 \ln |x| + C \)

(C) \( \frac{1}{2} x^2 - 3x - 10 \ln |x + 4| + C \)  
(D) \( \frac{1}{2} x^2 - 3x + 14 \ln |x + 4| + C \)
7. Ben has a rectangular fish tank that is 3 ft wide and 2 ft long. He fills the bottom (unevenly) with rocks and wants to know about how much space his fish has left to swim. The table below shows the depth of the water at \( \frac{1}{2} \) ft intervals from one end of the tank to the other.

<table>
<thead>
<tr>
<th>Position ((x))</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth ((h(x)))</td>
<td>3</td>
<td>2</td>
<td>3.5</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Which of the following estimates the amount of space that Ben’s fish has to swim using the Trapezoidal Rule with \( n = 4 \) applied to the integral \( V = \int_0^2 3h(x)dx \)?

(A) \( \frac{3}{4} \left( 3 + 2 \cdot 2 + 2 \cdot 3.5 + 2 \cdot 3 + 4 \right) \)

(B) \( \frac{1}{4} \left( 3 + 4 \cdot 2 + 2 \cdot 3.5 + 4 \cdot 3 + 4 \right) \)

(C) \( \frac{1}{4} \left( 3 + 2 \cdot 2 + 2 \cdot 3.5 + 2 \cdot 3 + 4 \right) \)

(D) \( \frac{3}{4} \left( 3 + 4 \cdot 2 + 2 \cdot 3.5 + 4 \cdot 3 + 4 \right) \)

8. Evaluate \( \lim_{x \to 0} \frac{e^{-x} + x - 1}{\cos(2x) - 1} \)

(A) \(-1\)  \hspace{1cm} (B) \(\frac{1}{4}\)  \hspace{1cm} (C) \(1\)  \hspace{1cm} (D) \(-\frac{1}{4}\)

9. Evaluate \( \int_1^\infty \frac{1}{x^4 + x^2} dx \)

(A) \(0\)  \hspace{1cm} (B) \(1\)  \hspace{1cm} (C) \(\frac{\pi}{4}\)  \hspace{1cm} (D) \(4-\frac{\pi}{4}\)

10. If \( f(t) = \int_0^t (x^2 + 1)^2 dx \), find \( f'(1) \)

(A) \(\frac{7}{3}\)  \hspace{1cm} (B) \(\frac{8}{3}\)  \hspace{1cm} (C) \(8\)  \hspace{1cm} (D) \(4\)

11. The area of the region bounded by the graphs of \( y = e^{2x} \) and \( y = \frac{x}{x^2+1} \) over the interval \( 0 \leq x \leq 1 \) equals

(A) \(\frac{e^2 - \ln(2) - 1}{2}\)

(B) \(e^2 - \ln(2) - 1\)

(C) \(\frac{e^2 + \ln(2) + 1}{2}\)

(D) \(2 \left( e^2 - \ln(2) - 1 \right)\)

12. A spring at rest is 6 feet long. Suppose the work required to stretch the spring to 9 feet in length is 4 foot-pounds. How much work is needed to stretch the spring from a length of 9 feet to a length of 13 feet? Leave your answer in foot-pounds.

(A) \(\frac{176}{3}\)  \hspace{1cm} (B) \(\frac{80}{3}\)  \hspace{1cm} (C) \(\frac{352}{9}\)  \hspace{1cm} (D) \(\frac{160}{9}\)
13. Integrate \( \int_0^1 e^x \tan(e^x) \, dx \)

(A) \( \sec^2(e) - \sec^2(1) \)  

(B) \( \ln \left( \frac{\cos(1)}{\cos(e)} \right) \)  

(C) \( \ln(\cos(e)) \)  

(D) \( -\cot(e) + \cot(1) \)

14. Let the region \( R \) be bounded by \( x = 2y + 6 \) and \( y^2 = x - 3 \). Set up the integral to compute the volume of the solid of revolution formed by revolving \( R \) about \( x = 14 \) using the disk/washer method. SET UP ONLY

(A) \( \pi \int_{-1}^{3} \left[ (14 - (2y + 6))^2 - (14 - (y^2 + 3))^2 \right] \, dy \)

(B) \( \pi \int_{-1}^{3} (11 - y^2)^2 - (8 - 2y)^2 \, dy \)

(C) \( 2\pi \int_{4}^{12} x \left( \sqrt{x - 3} - \frac{x - 6}{2} \right) \, dx \)

(D) \( 2\pi \int_{4}^{12} (14 - x) \left( \sqrt{x - 3} - \frac{x - 6}{2} \right) \, dx \)

15. Let the region \( R \) be bounded by \( x = 2y + 6 \) and \( y^2 = x - 3 \). Set up the integral to compute the volume of the solid of revolution formed by revolving \( R \) about \( y = 9 \) using the cylindrical shells method. SET UP ONLY

(A) \( 2\pi \int_{-1}^{3} y \left( (2y + 6) - (y^2 + 3) \right) \, dy \)

(B) \( 2\pi \int_{-1}^{3} y \left( (2y + 6)^2 - (y^2 + 3)^2 \right) \, dy \)

(C) \( 2\pi \int_{-1}^{3} (9 - y) \left( 2y - y^2 + 3 \right) \, dy \)

(D) \( 2\pi \int_{4}^{12} (y - 9) \left( (2y + 6) - (y^2 + 3) \right) \, dy \)