1. For \( f(x, y) = 3x^3 - y^3 - 9x + 12y + 3 \), which of the following statements is true?

1) \((1,1)\) is a saddle point
2) \((1,-2)\) is a local maximum
3) \((-1,2)\) is a local maximum
4) \((1,2)\) is a local minimum

2. For \( L = \lim_{(x,y) \to (0,0)} \frac{5xy}{x^2 + y^2} \), which of the following is true?

1) \( L = \infty \)
2) \( L = 0 \)
3) \( L \) does not exist
4) \( L = 5/2 \)

3. Let \( w = f(x, y) = xy^3 - e^{x^2} \) where \( x = u^2 + 3uv \) and \( y = 2e^v + u^2 \). The partial derivative \( \frac{\partial w}{\partial u} \) evaluated at \((u,v) = (1,0)\) is

1) \( 108 + 4e \)
2) \( 4e \)
3) \( 108 + e \)
4) \( 108 - 4e \)

4. Let the directional derivative of a function \( f(x, y) \) at a point \( P \) in the direction of \( \frac{\vec{i}}{\sqrt{5}} + \frac{\vec{j}}{\sqrt{5}} \) be \( \frac{16}{\sqrt{5}} \) and the partial derivative \( \frac{\partial f}{\partial x} \) evaluated at \( P \) be 6. Then, the directional derivative in the direction of \( \vec{i} - \vec{j} \) is

1) \( \frac{1}{\sqrt{2}} \)
2) \( 1 \)
3) \( \frac{1}{\sqrt{2}} \vec{i} - \frac{1}{\sqrt{2}} \vec{j} \)
4) \( \sqrt{40} \)

5. The Maclaurin series for \( f(x) = \frac{3}{1-2x} \) has the open interval of convergence

1) \((-1,1)\)
2) \((-\frac{1}{2}, \frac{1}{2})\)
3) \((-2,2)\)
4) \((2,4)\)

6. For a collection \( \{a_n\} \) of positive numbers which of the following combinations cannot occur

1) \( \sum_{n=1}^{\infty} a_n \) diverges and \( \sum_{n=1}^{\infty} (-1)^n a_n \) converges
2) \( \sum_{n=1}^{\infty} a_n \) converges and \( \sum_{n=1}^{\infty} (-1)^n a_n \) diverges
3) \( \lim_{n \to \infty} (-1)^n a_n \) does not exist and \( \lim_{n \to \infty} a_n \) exists
4) \( \lim_{n \to \infty} (-1)^n a_n \) exists and \( \lim_{n \to \infty} a_n \) exists

7. For positive numbers \( p, q, \) and \( r \), consider the three series

\[ \sum_{n=1}^{\infty} \frac{1}{n^p}, \sum_{n=1}^{\infty} \frac{1}{n^q}, \text{ and } \sum_{n=1}^{\infty} \frac{(-1)^n}{n^r}. \]
The collection of all positive values \( p, q, \) and \( r \) that make all three series converge is exactly:

1) \( p > 0, \ q > 0, \) and \( r > 1 \)  \hspace{1cm} 2) \( p > 1, \ q > 1, \) and \( r > 1 \)
3) \( p > 1, \ q > 1, \) and \( r > 0 \)  \hspace{1cm} 4) \( p > 1, \ q > 0, \) and \( r > 0 \)

8. The series \( \sum_{k=2}^{\infty} a_k \) has partial sums \( S_n = \sum_{k=2}^{n} a_k = \ln(n) \). Which of the following is true?

1) The series \( \sum_{k=2}^{\infty} a_k \) converges because \( \frac{d}{dx} \ln(x) = \frac{1}{x} \) satisfies \( \lim_{x \to \infty} \frac{1}{x} = 0 \)
2) The series \( \sum_{k=2}^{\infty} a_k \) converges because \( a_k = \ln(k) - \ln(k-1) = \ln\left(\frac{k}{k-1}\right) \) satisfies \( \lim_{k \to \infty} \ln\left(\frac{k}{k-1}\right) = 0 \)
3) The series \( \sum_{k=2}^{\infty} a_k \) diverges because \( \lim_{n \to \infty} S_n = \lim_{n \to \infty} \ln(n) = \infty \)
4) The series \( \sum_{k=2}^{\infty} a_k \) converges because for every \( p > 1 \), \( \lim_{n \to \infty} \frac{\ln(n)}{n^p} = 0 \)

9. Let \( I = \int_{0}^{e^{-0.01}} e^{-x^2} \, dx \). Which of the following is true?

1) \( I = \frac{e^{-0.01}}{0.2} \)  \hspace{1cm} 2) \( \frac{1}{10} < I < \frac{e^{0.01}}{10} \)  \hspace{1cm} 3) \( I = \sum_{k=1}^{\infty} \frac{1}{10^{2k}} \)  \hspace{1cm} 4) \( \frac{1}{10} - \frac{1}{900} < I < \frac{1}{10} \)

10. The average value of \( f(x,y) = xy^2 \) over the rectangle \( R \) with vertices \((0, -1), (2, -1), (2, 1), \) and \((0, 1)\) is

1) 0  \hspace{1cm} 2) 1/3  \hspace{1cm} 3) 2/3  \hspace{1cm} 4) 4/3

11. \( D \) is the planar region bounded by \( y = 0, \ y = x^2, \) and \( x = 1. \) The value of the integral \( \int \int_{D} x e^y \, dA \) is equal to

1) \( e - \frac{1}{2} \)  \hspace{1cm} 2) \( e - 1 \)  \hspace{1cm} 3) \( \frac{e}{2} - \frac{1}{2} \)  \hspace{1cm} 4) \( \frac{e}{2} - 1 \)

12. The volume of the solid enclosed by the planes \( x = 0, \ y = 0, \ z = 0, \) and \( x + y + z = 2 \) is represented by

1) \( \int_{0}^{2} \int_{0}^{2} \int_{0}^{2} \, dz \, dy \, dx \)  \hspace{1cm} 2) \( \int_{0}^{2} \int_{0}^{2-x} \int_{0}^{2-x-y} \, dz \, dy \, dx \)
3) \( \int_{0}^{2} \int_{0}^{2-x-y} \, dz \, dy \, dx \)  \hspace{1cm} 4) \( \int_{0}^{2-y} \int_{0}^{2-x} \int_{0}^{2-x-y} \, dz \, dy \, dx \)
13. The density of a thin plate is given by \( \rho(x, y) = xy \) and its mass by \( M \). The plate is bounded by \( y = \sqrt{x}, \ x = 1, \) and the \( x \)-axis. The \( y \)-coordinate of the plate’s center of mass is given by

\[
1) \quad \frac{1}{M} \int_0^1 \int_{y^2}^1 xy^2 \ dxdy \\
2) \quad \frac{1}{M} \int_0^1 \int_0^{y^2} x^2y \ dx \ dy \\
3) \quad \frac{1}{M} \int_0^1 \int_0^{y^2} xy^2 \ dx \ dy \\
4) \quad \frac{1}{M} \int_0^1 \int_{y^2}^{y^2} x^2y \ dx \ dy
\]

14. The volume of the solid inside the sphere \( x^2 + y^2 + z^2 = 4 \) and above the cone \( z = \sqrt{3x^2 + 3y^2} \) is given by

\[
1) \quad \int_0^{2\pi} \int_0^{\pi/6} \int_0^{\pi/2} \rho^2 \sin\phi \ d\rho \ d\phi \ d\theta \\
2) \quad \int_0^{2\pi} \int_{\pi/4}^{\pi/2} \int_0^{\pi/4} \rho^2 \sin\phi \ d\rho \ d\phi \ d\theta \\
3) \quad \int_0^{2\pi} \int_{\pi/3}^{\pi/2} \int_0^{\pi/4} \rho^2 \sin\phi \ d\rho \ d\phi \ d\theta \\
4) \quad \int_0^{2\pi} \int_{\pi/4}^{\pi/3} \int_0^{\pi/4} \rho^2 \sin\phi \ d\rho \ d\phi \ d\theta
\]