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1. Let $z = x^2e^y$, where $x = u^2v - 1$ and $y = uv - 2$. The partial derivative $\frac{\partial z}{\partial u}$ at $(u, v) = (1, 2)$ is

(1) 2 (2) 8 (3) 10 (4) 16

2. For the function $f(x, y) = x^3 - 12x + y^2$,

(1) there are no critical points.
(2) f has a saddle point at the critical point $(-2, 0)$.
(3) f has a local minimum at the critical point $(0, 0)$.
(4) f has a local maximum at the critical point $(-2, 0)$.

3. Let $f(x, y) = (1 + \sin(2x))y^3$. Using the total differential (or the linearization or standard linear approximation) of $f(x, y)$ at the point $P(0, 1)$, one can approximate the value of $(1 + \sin(0.02))(1.01)^3$ by

(1) 1.02 (2) 1.03 (3) 1.05 (4) 1.07

4. The tangent plane to the surface $z = xy^2$ at the point $P(1, 1, 1)$ contains the point $P(0, 0, k)$ when k is

(1) -3 (2) 0 (3) -2 (4) 5

5. At the point $P(1, 1, 1)$ the function

$$f(x, y, z) = 2x^2y + \frac{z^4}{y^2}$$

increases fastest in the direction of the vector

(1) $\mathbf{i} + \mathbf{k}$ (2) $\mathbf{i} + \mathbf{j} + \mathbf{k}$ (3) $4\mathbf{i}$ (4) $\mathbf{j} - \mathbf{k}$

6. The value of

$$\int_0^6 \int_{y/2}^3 e^{x^2}\, dx\, dy$$

is

(1) $e^9 - 1$ (2) $e^{36} - 1$ (3) $5e^9 - 1$ (4) $e^{18} - 1$

7. The area of the region that lies outside the cardioid $r = 1 + \sin\theta$ and inside the circle $r = 1$ is

$$(1)\ \int_0^{\pi}\int_1^{1+\sin\theta} r\,dr\,d\theta$$

$$(2)\ \int_0^{\pi}\int_{1+\sin\theta}^{1} r\,dr\,d\theta$$

$$(3)\ \int_{-\pi}^{0}\int_{1+\sin\theta}^{1} r\,dr\,d\theta$$

$$(4)\ \int_{\pi}^{2\pi}\int_1^{1+\sin\theta} r\,dr\,d\theta$$

8. The integral of $f(x,y,z) = z$ over the solid in the first octant bounded by the planes $y = 0$, $x = 0$, $z = 1$, and $z = x + y$ equals

$$(1)\ \int_0^1\int_0^1\int_0^1 z\,dx\,dy\,dz$$

$$(2)\ \int_0^1\int_0^1\int_{x+y}^1 z\,dz\,dx\,dy$$

$$(3)\ \int_0^1\int_0^{1-y}\int_{x+y}^1 z\,dz\,dx\,dy$$

$$(4)\ \int_0^1\int_0^{1-y}\int_0^{1-x-y} z\,dz\,dx\,dy$$

9. A solid has its volume given in cylindrical coordinates by

$$\int_0^{2\pi}\int_0^{2\sqrt{3}}\int_{r/\sqrt{3}}^{\sqrt{16-r^2}} r\,dz\,dr\,d\theta.$$

When re-expressed in spherical coordinates, this volume equals

$$(1)\ \int_0^{2\pi}\int_0^{\pi/3}\int_0^4 \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

$$(2)\ \int_0^{2\pi}\int_0^{\pi/3}\int_0^{4/\cos\phi} \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

$$(3)\ \int_0^{2\pi}\int_0^{\pi/6}\int_0^{16} \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

$$(4)\ \int_0^{2\pi}\int_0^{\pi/6}\int_{\sqrt{3}\cos\phi}^{16} \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

10. A thin plate is modeled by the planar region bounded by the lines $y = 3x$, $y + 3x = 6$, and the x-axis. The density of the plate at the point (x, y) is y, and the mass of the plate is 3. The y-coordinate of the plate's center of mass is

(1) $\frac{1}{3}\int_0^6 \int_{3x}^{6-3x} y\,dy\,dx$

(2) $\frac{1}{3}\int_0^3 \int_{y/3}^{(6-y)/3} y^2\,dx\,dy$

(3) $\frac{1}{3}\int_0^3 \int_{y/3}^{(6-y)/3} y\,dx\,dy$

(4) $\frac{1}{3}\int_0^3 \int_{y/3}^{(6-y)/3} xy\,dx\,dy$

11. Let A and B denote the series

$$A = \sum_{n=1}^{\infty} (-1)^n \frac{e^n}{n^e}, \qquad B = \sum_{n=1}^{\infty} \frac{1}{n^2 + \sqrt{n}}.$$

Then

(1) both series converge.
(2) both series diverge.
(3) A converges and B diverges.
(4) A diverges and B converges.

12. The series

$$\sum_{n=1}^{\infty} \frac{2^n + 1}{3^n}$$

(1) diverges.
(2) converges to 6.
(3) converges to 3.
(4) converges to $\frac{5}{2}$.

13. Suppose that the numbers a_n are positive numbers satisfying $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 3$. Then the open interval of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{a_n}{2^n} x^n$$

is

(1) $(-6, 6)$ (2) $(-\frac{1}{6}, \frac{1}{6})$ (3) $(-\frac{2}{3}, \frac{2}{3})$ (4) $(-\frac{3}{2}, \frac{3}{2})$

14. The first three nonzero terms of the Taylor series generated by $f(x) = \sin(x)$ with center at the point $a = \frac{\pi}{2}$ are

(1) $(x - \frac{\pi}{2}) - \frac{1}{3!}(x - \frac{\pi}{2})^3 + \frac{1}{5!}(x - \frac{\pi}{2})^5$

(2) $1 - \frac{1}{2}(x - \frac{\pi}{2})^2 + \frac{1}{4!}(x - \frac{\pi}{2})^4$

(3) $\frac{\pi}{2} - \frac{1}{3!}(\frac{\pi}{2})^3 + \frac{1}{5!}(\frac{\pi}{2})^5$

(4) $1 - \frac{1}{2}x^2 + \frac{1}{4!}x^4$

15. Suppose that the series $\sum_{k=1}^{\infty} a_k$ has partial sums $s_n = \sum_{k=1}^{n} a_k$ satisfying $s_n = 3 + \frac{1}{2^n}$. Then

(1) the series $\sum_{k=1}^{\infty} a_k$ diverges because $\lim_{n\to\infty} s_n = 3$.

(2) the series $\sum_{k=1}^{\infty} a_k$ converges to 4 because $\sum_{n=1}^{\infty} \frac{1}{2^n} = 1$.

(3) the series $\sum_{k=1}^{\infty} a_k$ diverges because $\lim_{k\to\infty} a_k \neq 0$.

(4) the series $\sum_{k=1}^{\infty} a_k$ converges to 3 because $\lim_{n\to\infty} s_n = 3$.